e1solns

Note Title 9/25/2006

MA 181 Exam 1 Solutions

1. a) $L = \int_0^{\pi} \sqrt{1 + [2\cos 2x]^2}\, dx$

b) $A = \int_0^{\pi} 2\pi\, |\sin(2x)| \sqrt{1 + [2\cos 2x]^2}\, dx$

2. a) $A = \frac{1}{2} |\vec{AB} \times \vec{AC}|$

$= \frac{1}{2} \left| \det \begin{bmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & 1 & 0 \\ 1 & 2 & 2 \end{bmatrix} \right|$

$= \frac{1}{2} | 2\hat{i} + 2\hat{j} - 3\hat{k} |$

$= \frac{1}{2} \sqrt{4 + 4 + 9} = \frac{\sqrt{17}}{2}$

b) $\vec{n} = \frac{\vec{AB} \times \vec{AC}}{|\vec{AB} \times \vec{AC}|} = \frac{2\hat{i} + 2\hat{j} - 3\hat{k}}{\sqrt{17}}$.

c) $\cos\theta = \dfrac{\vec{AC}\cdot\vec{AB}}{|\vec{AC}||\vec{AB}|} = \dfrac{(\hat{i}+2\hat{j}+2\hat{k})\cdot(-\hat{i}+\hat{j})}{\sqrt{1+4+4}\cdot\sqrt{1+1}}$

$$= \frac{1\cdot(-1)+2\cdot1+2\cdot0}{\sqrt{9}\,\sqrt{2}} = \frac{1}{3\sqrt{2}}$$

d) $\text{Proj}_{\vec{AB}}\vec{AC} = \dfrac{\vec{AC}\cdot\vec{AB}}{|\vec{AB}|^2}\vec{AB}$

$$= \frac{1}{(\sqrt{1+1})^2}(-\hat{i}+\hat{j}) = -\tfrac{1}{2}\hat{i}+\tfrac{1}{2}\hat{j}$$

3. $f'(t) = \int \sin(3t)\,dt = -\frac{1}{3}\cos 3t + C$

$f(t) = \int -\frac{1}{3}\cos 3t + C\ dt$

$$= -\frac{1}{9}\sin 3t + Ct + C_2$$

$f(0) = -\frac{1}{9}\underbrace{\sin 0}_{0} + C\cdot 0 + c_2 \overset{\text{want}}{=} 5$

So $c_2 = 5$ and we get

$$f(t) = -\frac{1}{9}\sin(3t) + Ct + 5$$

where C is an arbitrary const.

4. Cylindrical shells:

$$V = \int_0^{\pi/4} 2\pi x \left(\cos x - \sin x\right) dx$$

Washers:

$$V = \int_0^{\sqrt{2}/2} \pi \left(\sin^{-1} y\right)^2 dy + \int_{\sqrt{2}/2}^{1} \pi \left(\cos^{-1} y\right)^2 dy$$

5. Let $u = 3x + 2$. Then

$du = 3\,dx$ and $\boxed{dx = \frac{1}{3}du}$.

Also $x = \frac{1}{3}(u-2)$. So

$$\int x(3x+2)^{100}\,dx =$$

$$\int \frac{1}{3}(u-2)\cdot u^{100}\left(\frac{1}{3}du\right)$$

$$= \frac{1}{9}\int u^{101} - 2u^{100}\, du$$

$$= \frac{1}{9}\left[\frac{1}{102}u^{102} - \frac{2}{101}u^{101}\right] + C$$

$$= \frac{1}{9}\left[\frac{1}{102}(3x+2)^{102} - \frac{2}{101}(3x+2)^{101}\right] + C$$

6. $$V = \int_{-1}^{1} \pi (x^2 - c)^2\, dx$$

$$= \int_{-1}^{1} \pi (x^4 - 2x^2c + c^2)\, dx$$

$$= \pi\left[\frac{1}{5}x^5 - \frac{2}{3}x^3c + c^2x\right]_{-1}^{1}$$

$$= \pi\left[\frac{2}{5} - \frac{4}{3}c + 2c^2\right]$$

$$V(c) = \pi\left(2c^2 - \frac{4}{3}c + \frac{2}{5}\right)$$

$$V'(c) = \pi\left(4c - \frac{4}{3}\right)$$

$V'(c) = 0$ when $c = \frac{1}{3}$.

$$V(0) = \frac{2}{5}\pi$$

$$V\left(\frac{1}{3}\right) = \pi\left(\frac{2}{9} - \frac{4}{9} + \frac{2}{5}\right) = \left(\frac{2}{5} - \frac{2}{9}\right)\pi$$

$$V(1) = \pi\left(2 - \frac{4}{3} + \frac{2}{5}\right) = \left(\frac{2}{5} + \frac{1}{3}\right)\pi$$

Minimum is at $c = \frac{1}{3}$.