Solutions to Review Problems not done in class

1. Done in class.

2. $$\int \sqrt{\underbrace{y+1}_{u}}\,\underbrace{dy}_{du} = \int \frac{1}{x^2}\,dx$$

$$\frac{1}{\frac{1}{2}+1}(y+1)^{1/2+1} = -\frac{1}{x} + C$$

$$\frac{2}{3}(y+1)^{3/2} = -\frac{1}{x} + C$$

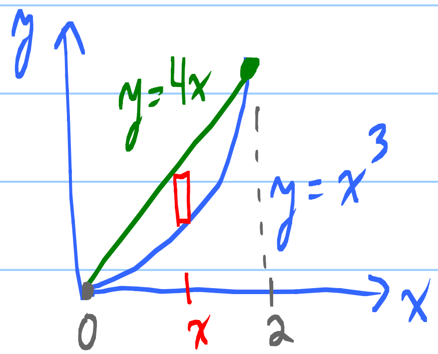
$y(-3) = 3$: $$\frac{2}{3}(3+1)^{3/2} = -\frac{1}{(-3)} + C$$

$$C = \frac{2}{3}\cdot 4^{3/2} - \frac{1}{3} = \frac{16}{3} - \frac{1}{3} = \frac{15}{3} = 5$$

So $$(y+1)^{3/2} = \frac{3}{2}\left[-\frac{1}{x} + 5\right]$$

$$y+1 = \left(\frac{15}{2} - \frac{3}{2x}\right)^{2/3}$$

$$\boxed{y = \left(\frac{15}{2} - \frac{3}{2x}\right)^{2/3} - 1}$$

3. 

$$V = \int 2\pi r h \Delta r = \int_0^2 2\pi x\left(4x - x^3\right)dx$$

$S' = S'_1 + S'_2$ where

$$S_1' = \int_0^2 2\pi x\sqrt{1+(4)^2}\,dx \quad \text{and}$$

$$S_2' = \int_0^2 2\pi x\sqrt{1+(3x^2)^2}\,dx$$

4. $$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{(3\sin^2 t)\cos t}{(-3\cos^2 t)(-\sin t)} = \frac{\sin t}{\cos t} = \tan t$$

$0 \le t \le 2\pi$. $\tan t = 1$ at $t = \frac{\pi}{4}$ and $\frac{5\pi}{4}$ only.

$$(x,y)_{t=\pi/4} = \left(-\cos^3\frac{\pi}{4}, \sin^3\frac{\pi}{4}\right) = \underline{\left(-\frac{1}{2^{3/2}}, \frac{1}{2^{3/2}}\right)}$$

$$(x,y)_{t=\frac{5\pi}{4}} = \left(-\cos^3\frac{5\pi}{4}, \sin^3\frac{5\pi}{4}\right) = \underline{\left(\frac{1}{2^{3/2}}, -\frac{1}{2^{3/2}}\right)}$$

5. Done in class.

6. $$\frac{2}{(x-1)(x^2+1)} = \frac{A}{x-1} + \frac{Bx+C}{x^2+1}$$

$$2 = A(x^2+1) + (Bx+C)(x-1)$$

$$2 = \underbrace{(A+B)}_{0}x^2 + \underbrace{(-B+C)}_{0}x + \underbrace{(A-C)}_{2}$$

$$\begin{cases} A+B=0 \\ -B+C=0 \\ A-C=2 \end{cases}$$

$C = A - 2$

$-B + (A-2) = 0$

$A - B = 2$

$$\begin{array}{rl} & \begin{cases} A+B=0 \\ A-B=2 \end{cases} \\ + & \hline \\ & 2A = 2 \end{array}$$

$\boxed{A=1}$ $\quad B=-A \quad \boxed{B=-1}$ $\quad C=B \quad \boxed{C=-1}$

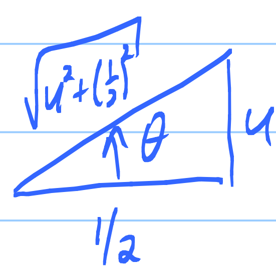
So $$\int \frac{2}{(x-1)(x^2+1)}\,dx = \int \frac{1}{x-1} - \frac{x}{x^2+1} - \frac{1}{x^2+1}\,dx$$

$$= \underline{\ln|x-1| - \tfrac{1}{2}\ln(x^2+1) - \tan^{-1}x + C}$$

$$\int \frac{dx}{\sqrt{4x^2-4x+2}} = \frac{1}{2}\int \frac{dx}{\sqrt{x^2-x+\frac{1}{2}}} = \frac{1}{2}\int \frac{dx}{\sqrt{(x-\frac{1}{2})^2+(\frac{1}{2})^2}}$$

Let $u = x - \frac{1}{2}$, Then $du = dx$.

$$I = \frac{1}{2}\int \frac{du}{\sqrt{u^2+(1/2)^2}}\,du$$



$u = \frac{1}{2}\tan\theta$

$$I = \frac{1}{2}\int \frac{\frac{1}{2}\sec^2\theta\,d\theta}{(\frac{1}{2}\sec\theta)}$$

$du = \frac{1}{2}\sec^2\theta\,d\theta$

$$= \frac{1}{2}\int \sec\theta \, d\theta = \frac{1}{2}\ln\left|\sec\theta + \tan\theta\right| + C$$

$$= \frac{1}{2}\ln\left|\frac{\sqrt{u^2+(\frac{1}{2})^2}}{(1/2)} + \frac{u}{(1/2)}\right| + C$$

$$= \frac{1}{2}\ln\left|2\sqrt{x^2-x+\frac{1}{2}} + 2\left(x-\frac{1}{2}\right)\right| + C$$

7. $$\int \underset{\substack{\uparrow \\ u}}{x}\, \underbrace{e^{-x}\,dx}_{dv} = uv - \int v\,du = x(-e^{-x}) - \int(-e^{-x})\,dx$$

$$= -xe^{-x} - e^{-x} + C$$

$du = dx \qquad v = \int e^{-x}\,dx = -e^{-x}$

$$\int_0^\infty xe^{-x}\,dx = \lim_{B\to\infty}\left[\left(-Be^{-B} - e^{-B}\right) - \left(-0\cdot e^{0} - e^{0}\right)\right]$$

$$= 0 - (-1) = \underline{\underline{1}}$$

8. Done in class.

9. $$A = \int_\alpha^\beta \frac{1}{2}r^2\,d\theta = \int_0^{\pi/2} \frac{1}{2}(\cos\theta + \sin\theta)^2\,d\theta$$

$$= \frac{1}{2}\int_0^{\pi/2} \cos^2\theta + 2\cos\theta\sin\theta + \sin^2\theta \;\, d\theta$$

$\cos^2\theta + \sin^2\theta = 1$ and $2\cos\theta\sin\theta = \sin 2\theta$.

So $A = \frac{1}{2}\int_0^{\pi/2} 1 + \sin 2\theta \, d\theta = \frac{1}{2}\left[\theta - \frac{1}{2}\cos 2\theta\right]_0^{\pi/2}$

$$= \frac{1}{2}\left(\left[\frac{\pi}{2} - \frac{1}{2}(-1)\right] - \left[0 - \frac{1}{2}\cdot 1\right]\right) = \underline{\underline{\frac{\pi}{4} + \frac{1}{2}}}$$

10. $[x^2 - 2x] + 4y^2 = 3$

$[(x-1)^2 - 1] + 4y^2 = 3$

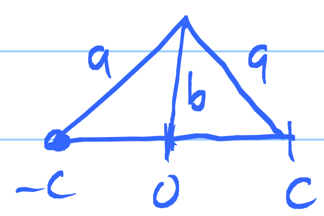
$(x-1)^2 + 4y^2 = 4$

$\frac{(x-1)^2}{2^2} + y^2 = 1$

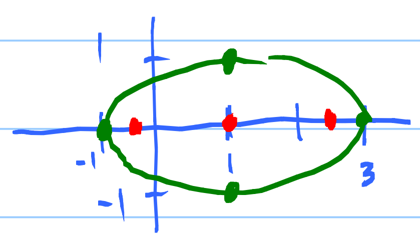
Ellipse centered at $(1,0)$.

$a = 2$

$b = 1$



$c = \sqrt{a^2 - b^2} = \sqrt{4-1} = \sqrt{3}$



Foci: $1 \pm \sqrt{3}$

11, 12, 13. Done in class.

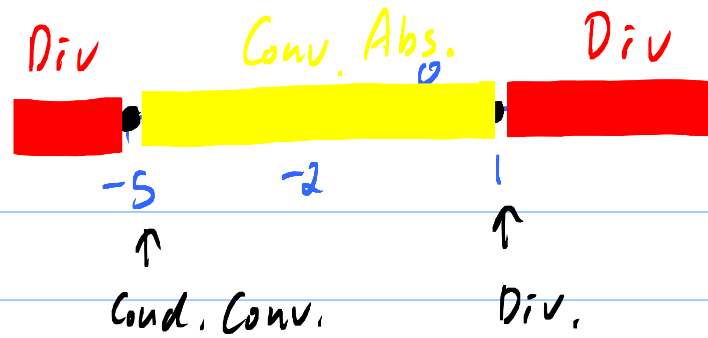
14. $1 + \sum_{n=1}^{\infty} \frac{(x+2)^n}{3^n \cdot n}$

Ratio Test: $\left|\frac{u_{n+1}}{u_n}\right| =$

$\frac{n}{n+1} \cdot \frac{|x+2|}{3} \longrightarrow \frac{|x+2|}{3}$ < 1 Conv. Abs.

> 1 Diverge

$$\begin{cases} |x+2| < 3 & \text{Conv. Abs.} \\ |x+2| > & \text{Div.} \end{cases}$$



Endpoints: $x = -5, 1$.

At $x=-5$: $1 + \sum_{n=1}^{\infty} \frac{(-3)^n}{3^n n} = 1 + \sum_{n=1}^{\infty} \frac{(-1)}{n}$

Converges Conditionally.

At $x=1$: $1 + \sum_{n=1}^{\infty} \frac{3^n}{3^n n} = 1 + \sum_{n=1}^{\infty} \frac{1}{n}$

Diverges.

15. $\frac{1}{x^2-1} = \frac{1/2}{x-1} - \frac{1/2}{x+1}$ $\quad L = \sum_{n=2}^{\infty} \frac{1}{n^2-1} = \frac{1}{2}\sum_{n=2}^{\infty}\left(\frac{1}{n+1} - \frac{1}{n-1}\right)$

$$= \frac{1}{2}\left[\left(\frac{1}{3}-\frac{1}{1}\right) + \left(\frac{1}{4}-\frac{1}{2}\right) + \left(\frac{1}{5}-\frac{1}{3}\right) + \left(\frac{1}{6}-\frac{1}{4}\right) + \left(\frac{1}{7}-\frac{1}{5}\right) + \cdots\right]$$

Everything cancels except $-\frac{1}{1}$ and $-\frac{1}{2}$. $L = \frac{1}{2}\left(-1-\frac{1}{2}\right) = -\frac{3}{4}$

16. Done in class.