solutions_2

Note Title 11/14/2006

MATH 181 Exam 2 solutions

1. a) $\dfrac{2x+7}{(x+1)(x-4)} = \dfrac{-1}{(x+1)} + \dfrac{3}{x-4}$

So $\underline{I} = \int \frac{-1}{x+1} + \frac{3}{x-4}\, dx = -\ln|x+1| + 3\ln|x-4| + C$

1. b) $I = \int \dfrac{2x+7}{(x-1)^2+2^2}\, dx$ $\quad u = x-1$, $du = dx$ $\quad \boxed{x = u+1}$

$= \int \dfrac{2(u+1)+7}{u^2+2^2}\, du = \int \dfrac{2u\,du}{\underbrace{u^2+2^2}_{v}} + 9\int \dfrac{1}{u^2+2^2}\, du$

$= \int \dfrac{dv}{v} + 9 \cdot \frac{1}{2} \operatorname{Tan}^{-1} \frac{u}{2} + C$

$= \operatorname{Ln}|v| + \frac{9}{2} \operatorname{Tan}^{-1}\left(\frac{x-1}{2}\right) + C$

$= \operatorname{Ln}\left[(x-1)^2+4\right] + \frac{9}{2} \operatorname{Tan}^{-1}\left(\frac{x-1}{2}\right) + C$

1. c) $u = \ln(x+2)$ $\qquad dv = 2x\, dx$

$du = \frac{1}{x+2}\, dx$ $\qquad v = \int 2x\, dx = x^2$

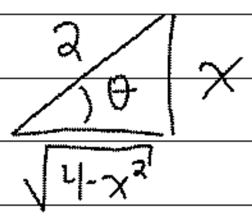
$$\int \underbrace{\ln(x+2)}_{u}\,\underbrace{(2x\,dx)}_{dv} = uv - \int v\,du$$

$$= \left(\ln(x+2)\right)x^2 - \int x^2 \cdot \frac{1}{x+2}\,dx$$

$$= x^2 \ln(x+2) - \int \underbrace{\frac{x^2}{x+2}}_{x-2+\frac{4}{x+2}\ \text{by division}}\,dx$$

$$= x^2 \ln(x+2) - \frac{x^2}{2} + 2x - 4\ln(x+2) + C$$

1. d)



$x = 2\sin\theta \qquad dx = 2\cos\theta\,d\theta$

$\sqrt{4-x^2} = 2\cos\theta$

$$I = \int \frac{\left(\sqrt{4-x^2}\right)^3}{x^6}\,dx = \int \frac{(2\cos\theta)^3}{(2\sin\theta)^6}\,(2\cos\theta\,d\theta)$$

$$= \frac{1}{4}\int \frac{\cos^4\theta}{\sin^6\theta}\,d\theta$$

2. $\int \frac{dy}{\sqrt{y^2+1}} = \int 2x\,dx$

$\sinh^{-1} y = x^2 + C \leftarrow$ $y=0$ when $x=0$: $\sinh^{-1}(0) = 0^2 + C$

$0 = C$

So $\sinh^{-1} y = x^2$

$y = \sinh(x^2)$

3. a) $\lim_{x\to 0+} \frac{(\ln x)^2}{\left(\frac{1}{x}\right)} \overset{L'H}{=} \lim_{x\to 0+} \frac{2(\ln x)\cdot\frac{1}{x}}{\left(\frac{-1}{x^2}\right)}$

$= \lim_{x\to 0+} \frac{2\ln x}{\left(\frac{-1}{x}\right)} \overset{L'H}{=} \lim_{x\to 0+} \frac{2\cdot\frac{1}{x}}{\left(\frac{1}{x^2}\right)}$

$= \lim_{x\to 0^+} 2x = \underline{\underline{0}}$

3. b) $y = (\ln x)^{1/x}$ $\qquad \ln y = \frac{1}{x}\ln(\ln x)$

$\lim_{x\to\infty} \frac{\ln(\ln x)}{x} \overset{L'H}{=} \lim_{x\to\infty} \frac{\frac{1}{\ln x}\cdot\frac{1}{x}}{1} = 0$

$$y = e^{\ln y} \longrightarrow e^0.$$

So $\lim_{x \to \infty} (\ln x)^{1/x} = e^0 = \underline{\underline{1}}$

4. $T(1) = 200\, e^{-K \cdot 1} = 100$

$$e^{-K} = \frac{100}{200} = \frac{1}{2}$$

$$-K = \ln e^{-K} = \ln \tfrac{1}{2} = \ln 2^{-1} = -\ln 2$$

So $K = \ln 2$.

$$T(3) = 200\, e^{-(\ln 2)\cdot 3} = 200\, e^{-\ln 2^3} = 200\, e^{\ln \frac{1}{8}} = \frac{200}{8} = \underline{\underline{25}}$$

5. $a^b = e^{b \ln a}$ if $a > 0$.

So $x^{(2x)} = e^{2x \ln x}$

$$(x^2)^x = e^{x \ln x^2} = e^{x(2 \ln x)} = \checkmark$$