

MATH 425, Exam I

(10) **1.** Prove that a *real valued* analytic function on the unit disc must be constant.

(20) **2.** Compute

$$\int_{\gamma} \frac{1}{z} dz$$

where γ is any curve in the plane that starts at $1 + i$ and ends at $-1 + i$ and that avoids the set $\{z : z = it, t \geq 0\}$ (i.e., the positive imaginary axis, including $z = 0$).

(20) **3.** Multiply

$$1 + z + z^2 + \cdots + z^N$$

by $(1 - z)$ to obtain a famous formula for the sum. Explain how to use the formula to find a short expression for

$$1 + \cos \theta + \cos 2\theta + \cdots + \cos N\theta.$$

(You do not have to simplify the expression.)

(20) **4.** What is the image of the right half plane $\{z : \operatorname{Re} z > 0\}$ under the mapping $f(z) = \frac{z}{z+2i}$? How would you define an analytic square root of $f(z)$ on the right half plane?

(20) **5.** Let γ be the curve segment described by the complex valued function $z(t) = 2e^{it}$ for $0 \leq t \leq \pi$. Let

$$F(z) = \int_{\gamma} \frac{1}{w - z} dw.$$

Prove that

$$\lim_{z \rightarrow \infty} F(z) = 0.$$

(10) **6.** Show that

$$|e^{(z^2)}| \leq e^{|z^2|}$$

and identify conditions for equality to hold.