

MATH 425, Exam II

- (20) **1.** Let C denote the unit circle parameterized in the counterclockwise direction. Compute

$$\int_C \frac{e^{2z}}{z^4} dz.$$

Explain.

- (20) **2.** Show that an entire function that takes no values in the line segment $[-1, 1]$ along the real line must be constant.

- (20) **3.** Find the radius of convergence of the power series

$$\sum_{n=1}^{\infty} \frac{n^n}{n!} z^{n^2}.$$

- (20) **4.** Find the first three terms in the Taylor series for the function $\frac{1+z}{1-z}$ about $z = i$. What is the radius of convergence of the series? Explain.

- (20) **5.** Let C_R denote the piece of a circle parameterized by $z(t) = Re^{it}$ for $0 \leq t \leq \pi/8$. Show that

$$\int_{C_R} e^{-z^2} dz$$

tends to zero as R tends to infinity.