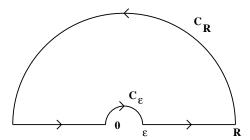**Math 425**
Exam 2
Each problem is worth 25 points

1. Use the contour pictured below to compute

$$\int_0^\infty \frac{\operatorname{Ln} x}{(x^2 + 4)^2}\, dx.$$

Define the branch of a complex logarithm that you use and justify your calculations and limits.

2. Assume that f is analytic on $D_1(0) - \{0\}$ and satisfies the estimate

$$|f(z)| \le \frac{C}{|z|^\alpha}$$

there for some constant $C > 0$ and constant α with $0 < \alpha < 1$. Prove that f has a removable singularity at $z = 0$. Hint: Consider the type of the singularity of $F(z) = zf(z)$ at the origin.

3. Find an analytic function that maps $\{z : 0 < \operatorname{Re} z < 1\}$ one-to-one onto the first quadrant.

4. Prove that there are no polynomials of the form

$$P(z) = z^n + a_{n-1}z^{n-1} + \cdots + a_1 z + a_0$$

satisfying $|P(z)| < 1$ when $|z| = 1$. Hint: Rouché's