MATH 425, Practice Problems

1. Prove that if an analytic function f has constant modulus on a domain, then it must be constant there.

2. Let γ denote any curve that starts at 2 and ends at -2 and avoids the set $\{z : \operatorname{Im} z \leq 0\}$. Compute

a) $\displaystyle\int_\gamma \frac{1}{z^3}\,dz$

b) $\displaystyle\int_\gamma \frac{1}{z}\,dz$

3. Compute

$$\int_0^\pi e^{3it}\,dt$$

where t is a real variable.

4. Prove the following version of the Chain Rule. If f is analytic on a domain Ω and $z(t)$ is a complex valued function of the real variable t that is differentiable and maps into Ω, then $h(t) = f(z(t))$ is differentiable and $h'(t) = f'(z(t))z'(t)$.

5. Find all z so that $\sin z = 2$.

6. Find a conformal mapping that maps the strip $\{z : 0 < \operatorname{Re} z < 1\}$ one-to-one onto the unit disc.

7. Let C_R denote the half circle parameterized by $z(t) = Re^{it}$ for $0 \leq t \leq \pi$. Show that

$$\int_{C_R} \frac{1}{z^4+1}\,dz$$

tends to zero as R tends to infinity.

8. Find an analytic function that maps the quarter disc

$$\{re^{i\theta} : 0 < r < 1, \quad 0 < \theta < \pi/2\}$$

one-to-one onto the strip

$$\{z : 0 < \text{Im } z < 1\}.$$

9. What is the image of the unit disc under the linear fractional transformation $T(z) = z/(1-z)$?