517. 7. First, I will do 11.7:5.

$$\int_0^\infty \frac{\cos(\pi w/2)}{1-w^2} \cos xw \, dw = \begin{cases} \frac{\pi}{2}\cos x, & 0 \le x \le \frac{\pi}{2} \\ 0, & x \ge \frac{\pi}{2} \end{cases}$$

Fourier cosine Integral

$$\int_0^\infty A(w) \cos xw \, dw = f(x)$$

where $A(w) = \frac{2}{\pi}\int_0^\infty f(v) \cos wv \, dv$

$$= \frac{2}{\pi}\int_0^{\pi/2} \frac{\pi}{2} \cos v \cos wv \, dv$$

$$\cos\alpha \cos\beta = \frac{1}{2}\cos(\beta-\alpha) + \frac{1}{2}\cos(\alpha+\beta)$$

$$\int_0^{\pi/2} \frac{1}{2}\cos v(w-1) + \frac{1}{2}\cos v(w+1) \, dv$$

$$= \frac{1}{2}\left[\frac{1}{w-1}\sin v(w-1) + \frac{1}{w+1}\sin v(w+1)\right]_{v=0}^{\pi/2}$$

$$= \frac{1}{2}\left(\frac{1}{w-1}\sin\left(\frac{\pi}{2}w - \frac{\pi}{2}\right) + \frac{1}{w+1}\sin\left(\frac{\pi}{2}w + \frac{\pi}{2}\right)\right)$$

But $\sin\left(\frac{\pi}{2}w - \frac{\pi}{2}\right) = -\cos\left(\frac{\pi}{2}w\right)$ and

$\sin\left(\frac{\pi}{2}w + \frac{\pi}{2}\right) = \cos\left(\frac{\pi}{2}w\right)$.

So $A(w) = \frac{1}{2}\cos\frac{\pi w}{2}\left[\frac{-1}{w-1} + \frac{1}{w+1}\right] =$

$$= \frac{\cos \pi w/2}{1-w^2}$$

Now that we know problem 5 is true, the rest is easy.

$$\underbrace{\sqrt{\frac{2}{\pi}}\int_0^\infty \frac{\cos(\pi w/2)}{1-w^2}\cos wx\, dw}_{\tilde{\mathcal{F}}_c\left[\frac{\cos(\pi w/2)}{1-w^2}\right](x)} = \sqrt{\frac{2}{\pi}}\, f(x)$$

dummy var in $\int$ is w and x is the transform var.

So $$\tilde{\mathcal{F}}_c\left[\frac{\cos(\pi x/2)}{1-x^2}\right](w) = \sqrt{\frac{2}{\pi}}\, f(w)$$

$$= \sqrt{\frac{2}{\pi}} \cdot \begin{cases} \frac{\pi}{2}\cos w & 0 \le w \le \frac{\pi}{2} \\ 0 & w \ge \frac{\pi}{2} \end{cases}$$

$$= \begin{cases} \sqrt{\frac{\pi}{2}}\cos w & 0 \le w \le \frac{\pi}{2} \\ 0 & w \ge \frac{\pi}{2} \end{cases}$$