Solutions to graded problems, HWK 11

p. 542: 8.
\[
\frac{\partial}{\partial t} (e^{-9t} \sin \omega x) = c^2 \frac{\partial^2}{\partial x^2} (e^{-9t} \sin \omega x)
\]

\[-9e^{-9t} \sin \omega x = c^2 (-\omega^2 e^{-9t} \sin \omega x)\]

Yes, if \(-9 = -c^2 \omega^2\), i.e., \(c\omega = 3\). (Not grading graphs)

p. 542: 10.
\[
\frac{\partial^2}{\partial x^2} (e^x \cos y) + \frac{\partial^2}{\partial y^2} (e^x \cos y) = 0
\]

\[e^x \cos y + e^x (-\cos y) = 0 \checkmark\]

\[
\frac{\partial^2}{\partial x^2} (e^x \sin y) + \frac{\partial^2}{\partial y^2} (e^x \sin y) = 0 \checkmark
\]

\[e^x \sin y + e^x (-\sin y) = 0 \checkmark\]

Not grading graphs, but this MAPLE command does the job nicely:
with (plots);

\[\text{plot3d}(\exp(x) \times \cos(y), x=-3..3, y=-2 \times \Pi..2 \times \Pi)\];

p. 542: 19.
\[
\frac{du}{dy} + y^2 u = 0
\]

\[\frac{du}{dy} = -y^2 u \leftarrow \text{separable in } y\]

\[\text{treat like ODE in } y \text{ with parameter } x \text{ floating along.}\]
\[ \frac{du}{u} = -y^2 \, dy \]
\[ \int \frac{du}{u} = -\int y^2 \, dy \]

\[ \ln |u| = -\frac{1}{3} y^3 + C(x) \]
\[ |u| = \exp \left( -\frac{1}{3} y^3 + C(x) \right) = e^{C(x)} e^{-\frac{1}{3} y^3} \]

\[ u = \pm e^{C(x)} e^{-\frac{1}{3} y^3} = K(x) e^{-\frac{1}{3} y^3} \]

where \( K(x) \) is an arbitrary function of \( x \).

You can also use the method I showed in class to solve a first order linear ODE.

\[ u(x,t) = \sum_{n=1}^{\infty} c_n \sin n\pi x \cos n\pi t \]

where \( c_n = \frac{2}{L} \int_{0}^{L} f(x) \sin n\pi x \, dx = \frac{2}{n\pi^2} \int_{\frac{3}{4}}^{\frac{5}{4}} (x-\frac{3}{4}) \sin n\pi x \, dx + \frac{2}{n\pi^2} \int_{-\frac{1}{4}}^{\frac{1}{4}} (-x+\frac{3}{4}) \sin n\pi x \, dx \]

\[ = -\frac{2}{n^2 \pi^2} \left( \sin \frac{n\pi}{4} - 2 \sin \frac{n\pi}{2} + \sin \frac{3n\pi}{4} \right) \]

via integration by parts and algebra.

To graph, use the odd periodic extension:

\[ f(x) \]

\[ x \in [-3, 3] \]
Make two copies. \( f^*(x+t) \) starts like \( f^* \) and moves left at a speed of one. \( f^*(x-t) \) starts like \( f^* \) and moves right at a speed of one. The superposition \( \frac{1}{2}(f^*(x+t)+f^*(x-t)) \) is the solution. I will demonstrate how to animate this in MAPLE.

**p. 556: 8.** Need the odd periodic extension:

\[
-3 \quad -1 \quad 0 \quad 1 \quad 3
\]

\[
\frac{1}{2}(f^*(x+t)+f^*(x-t))
\]

Same technique as above to animate.

I will demonstrate how in class.