$[-\pi, \pi]$

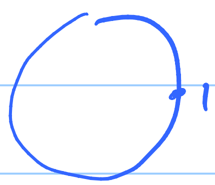
$$f(x) = a_0 + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx)$$

$$\frac{1}{\pi}\int_{-\pi}^{\pi} f(x)^2\,dx = 2a_0^2 + \sum_{n=1}^{\infty}(a_n^2 + b_n^2)$$

$$x = \sum_{n=1}^{\infty} b_n \sin nx$$

$$b_n = \frac{2(-1)^{n+1}}{n}$$

$$\frac{1}{\pi}\int_{-\pi}^{\pi} x^2\,dx = \sum_{n=1}^{\infty}\left(\frac{2(-1)^{n+1}}{n}\right)^2 = 4\sum_{n=1}^{\infty}\frac{1}{n^2}$$

28. 

$\Delta u = 0 \leftarrow$ <u>u harmonic</u>

$$u(1,\theta) = \underline{2 + 5\cos 3\theta + 7\sin 5\theta}$$

$$u(r,\theta) = a_0 + \sum \left(a_n \left(\frac{r}{R}\right)^n \cos n\theta + b_n \left(\frac{r}{R}\right)^n \sin n\theta\right)$$

$R = 1$

$$u(1,\theta) = \text{Fourier Series} \stackrel{\text{want}}{=} f(\theta)$$

$$\boxed{u(r,\theta) = 2 + 5r^3\cos 3\theta + 7r^5\sin 5\theta}$$

28. $f(\theta) = \cos 2\theta \leftarrow$ $a_2 = 1$

other a's, b's $= 0$

$$u(r,\theta) = \underline{r^2 \cos 2\theta}$$

$\mathbb{A}$ diagonalizable? $\mathbb{A}$ $n \times n$.

$\mathbb{A}$ symmetric? yes. [Can get orthonormal set of e-vects: $\mathbb{P} = [\text{e-vects}]$, $\mathbb{P}^{-1} = \mathbb{P}^T$]

Find e-vals. $\underline{n \text{ distinct e-vals}}$? yes.

some repeat: but get full set of lin. indep e-vects. yes.

$$\mathbb{P} = [\text{e-vects as cols}]$$

$\updownarrow$ same order

$$\mathbb{D} = \begin{bmatrix} \lambda_1 & & \\ & \ddots & \\ & & \lambda_n \end{bmatrix}$$

$$\mathbb{A} = \mathbb{P}^{-1} \mathbb{D} \mathbb{P}$$

$\mathbb{A}$ symm., choosing e-vects orthonormal $\mathbb{P}^{-1} = \mathbb{P}^T$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 2 \\ 0 & -3 & 4 \end{bmatrix} \qquad 1, 1, 2$$

$\underline{\lambda = 1}$: $(\mathbb{A} - \lambda \mathbb{I})\vec{a} = 0$

a_2 bound

$$\left[\begin{array}{ccc|c} 0 & 0 & 0 & 0 \\ 0 & -2 & 2 & 0 \\ 0 & -3 & 3 & 0 \end{array}\right] \sim \left[\begin{array}{ccc|c} 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$\uparrow$ a_1 $\qquad$ $\uparrow$ a_3 free

A is nondefective. E_1 is 2-dim^l

Wave } homog bndry cond — Wave, String prob.
Heat } — Heat, Iced ends

Wave: $u(x,t) = \sum_{n=1}^{\infty} \sin\frac{n\pi x}{L}\left(A_n \cos\frac{cn\pi t}{L} + B_n \sin\frac{cn\pi t}{L}\right)$

Heat: $\sum_{n=1}^{\infty} A_n \sin\frac{n\pi x}{L}\, e^{-c^2\left(\frac{n\pi}{L}\right)^2 t}$

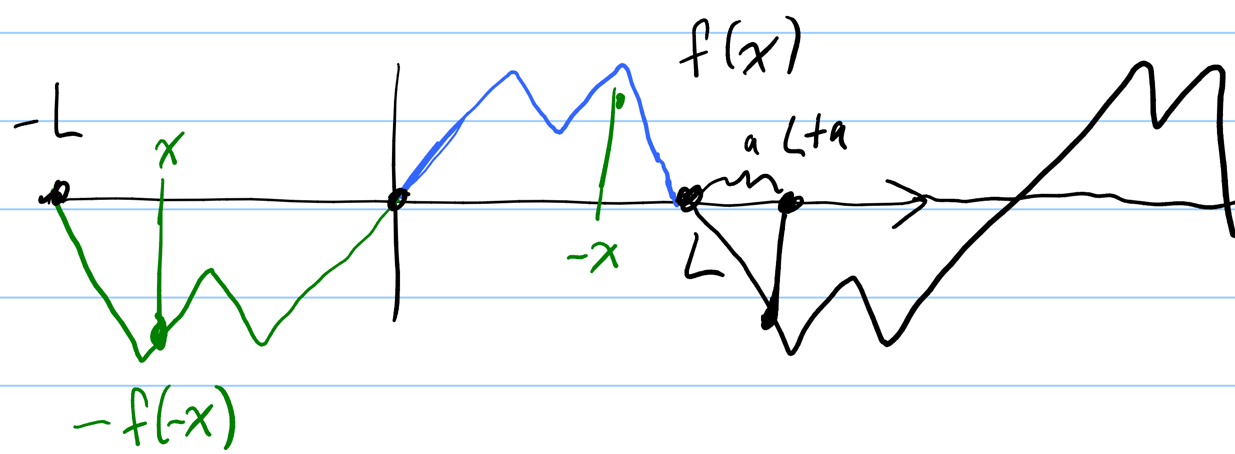
$$u(x,0) = \sum_{n=1}^{\infty} A_n \sin\frac{n\pi x}{L} = f(x)$$

F. Sine Series

$$\frac{\partial u}{\partial t}(x,0) = \sum_{n=1}^{\infty} \frac{cn\pi}{L} B_n \sin\frac{n\pi x}{L} = g(x)$$

F. Sine Series for g

$$B_n = \frac{1}{\left(\frac{cn\pi}{L}\right)} \cdot \frac{2}{L}\int_0^L g(x) \sin\frac{n\pi x}{L}\, dx$$



Fourier Sine Series conv. on $[0, L]$ to $f(x)$

Outside $[0, L]$, need <u>odd</u> periodic ext.

D'Alembert's $u(x,t) = \frac{1}{2}\left[f(x+ct) + f(x-ct)\right]$

odd periodic ext from $[0, L]$

$$\begin{bmatrix} 1 & 2 & 3 & \cdots \\ 7 & 8 & 9 & \cdots \end{bmatrix} \sim \begin{bmatrix} 1 & 2 & \cdots \\ 0 & 1 & \cdots \end{bmatrix}$$

Basis for $\underbrace{\text{col space}}_{\subset \mathbb{R}^2}$ $\begin{pmatrix} 1 \\ 7 \end{pmatrix}, \begin{pmatrix} 2 \\ 8 \end{pmatrix}$

Col Space $= \mathbb{R}^2$. $\begin{pmatrix} 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \end{pmatrix}$ is also a basis

$$\begin{bmatrix} 1 & 7 \\ 2 & 8 \\ \cdots & \cdots \\ \cdots & \cdots \end{bmatrix} \sim \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

<u>Two</u> lin indep vectors in $\mathbb{R}^2$ form a basis.

n . . . . . . $\mathbb{R}^n$ . . . . . .

$$u(x,t) = \varphi(x+ct) + \psi(x-ct)$$

$$\boxed{= \frac{1}{2}\left[\underset{\sin(x+2t)}{f(x+ct)} + \underset{\sin(x-2t)}{f(x-ct)}\right] + \frac{1}{2c}\int_{x-ct}^{x+ct} g(u)\,du}$$

$c=2$

$$\frac{1}{2\cdot 2}\int_{x-2t}^{x+2t} \sin 2u\,du$$

$$u(x,t) = \frac{1}{2}\left[\sin(x+2t) + \sin(x-2t)\right] + \underbrace{\frac{1}{4}\left[-\frac{1}{2}\cos 2u\right]_{x-2t}^{x+2t}}_{}$$

$$-\frac{1}{8}\left[\cos 2(x+2t) - \cos 2(x-2t)\right]$$

Wave Eqn: D'Alembert's works even for ∞ string.

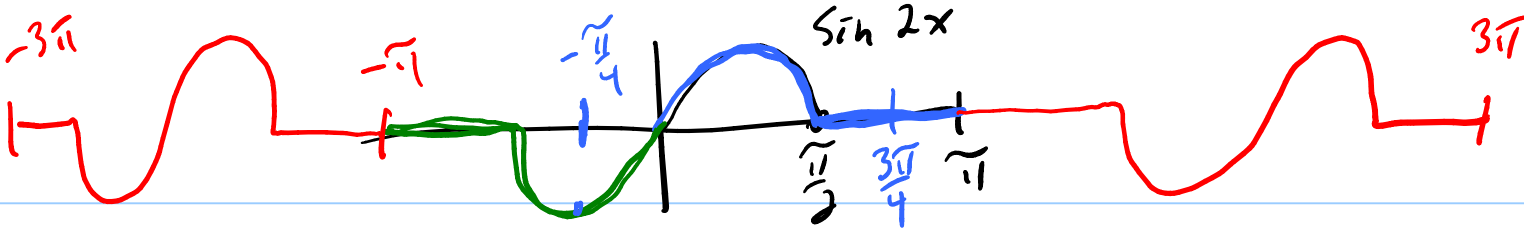
23.

$$f(x) = \sum_{n=1}^{\infty} A_n \sin nx$$

given

$$u(x,t) = \sum_{n=1}^{\infty} A_n \sin nx \cos cnt$$

safe way

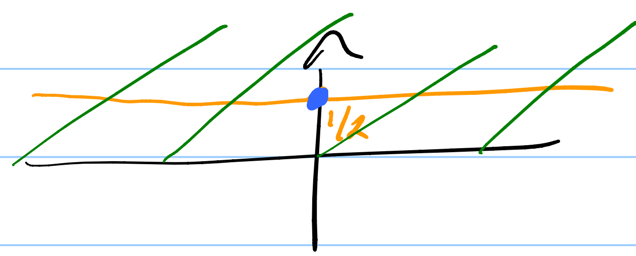


f^* odd periodic ext.

$$u\left(\frac{\pi}{4},\frac{\pi}{2}\right) = \frac{1}{2}\left[f^*\left(\frac{\pi}{4}+\frac{\pi}{2}\right) + f^*\left(\frac{\pi}{4}-\frac{\pi}{2}\right)\right]$$

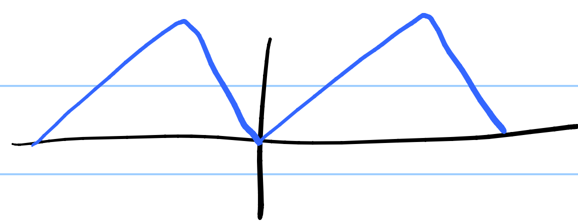
$$= \frac{1}{2}\left[\underbrace{f^*\left(\frac{3\pi}{4}\right)}_{0} + \underbrace{f^*\left(-\frac{\pi}{4}\right)}_{-1}\right] = -\frac{1}{2}$$

18. $f(x) = \frac{1}{2} - \sum \underbrace{(\text{sines})}_{\text{odd}}$

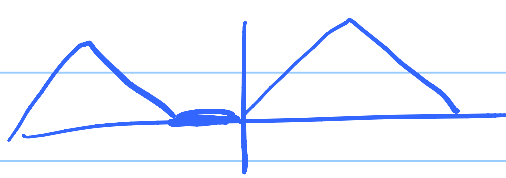
C. 

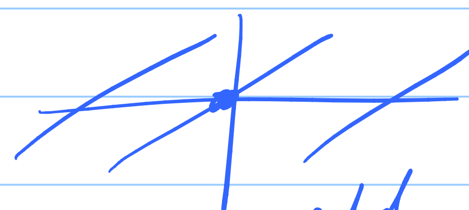
19. $\underbrace{\frac{1}{2}}_{a_0} + \sum \overset{a_n}{\underbrace{(\text{cosines})}_{\text{even}}}$ ← even

Cosine series are even

A. 

even

B. 

D. 

odd

HWK 12, more sol^{ns} on H. Page.