**Math 527 Fall 2008**
**Midterm 2**
**Nov 17, 2008**

**Faculty: B. Kaufmann**

Name:

Signature:

Student ID Number:

**Directions:** Work on as many of the problems below as you can. Start by working in the space below the problem; if this is not sufficient use the back of another page and clearly mark that the problem is continued elsewhere. Do not take the exam apart. **Be sure to show all your work!**

There are five problems. The points allotted to each problem are given in parenthesis. The total score is 100.

NO CALCULATORS, BOOKS, OR PAPERS ARE ALLOWED. Use the back of the test pages for scrap paper.

A table of Laplace transforms is given on the next page for your convenience.

| Problem | Max. | Points |
|---|---|---|
| 1 | 15 | |
| 2 | 15 | |
| 3 | 24 | |
| 4 | 22 | |
| 5 | 24 | |
| Total | 100 | |

**Problem 1:** *[15 points]* Compute the Laplace transform of

$$f(t) = t\,u(t-\pi)$$

**Problem 2:** *[15 points]* Find the inverse Laplace transform of

$$F(s) = \frac{3\,s\,e^{-5s}}{s^2 - 2s + 2}$$

**Problem 3:** Let $f(x)$ be the function defined on $0 < x < \pi$ as

$$f(x) = \begin{cases} 1 & \text{for} \quad 0 < x < \frac{\pi}{2} \\ \frac{1}{2} & \text{for} \quad \frac{\pi}{2} < x < \pi \end{cases}$$

a) *[6 points]* Sketch the even periodic extension of $f(x)$ for $-\pi < x < \pi$.

b) *[6 points]* Sketch the odd periodic extension of $f(x)$ for $-\pi < x < \pi$.

c) *[12 points]* Calculate the first two terms of the Fourier cosine series of $f(x)$ (i.e. $a_0$ and $a_1$). Which periodic extension of $f(x)$ does the Fourier cosine series describe?

**Problem 4:**

a) *[14 points]* Calculate the complex Fourier series of $f(x) = e^{2x}$ if $-\pi < x < \pi$ and $f(x + 2\pi) = f(x)$.

b) *[8 points]* Find the real Fourier series from your answer in a).

**Problem 5:** Let $f(x)$ be the function defined as

$$f(x) = \begin{cases} \frac{\pi}{2} & \text{for} \quad 0 < x < 1 \\ 0 & \text{for} \quad x > 1 \end{cases}$$

a) *[14 points]* Compute the Fourier sine transform of $f(x)$

b) *[10 points]* Using your answer from part a), evaluate the integral

$$\int_0^\infty \frac{(1 - \cos\omega)\sin\frac{\omega}{2}}{\omega} d\omega$$