Sample for the Midterm 2 for MA527

1. Find the inverse Laplace transform of $\frac{s^2+6}{(s-2)(s^2+2s+2)}$.

2. Find the inverse Laplace transform of $\frac{e^{-3s}}{s^2+4s+5}$.

3. Compute $u(t-1) * (e^{-2t}u(t))$ and its Laplace transform.

4. Solve $y(t) = 2t - 4\int_0^t y(\tau)(t-\tau)\mathrm{d}\tau$.

5. Solve $y'' + 2y' - 3y = 8e^{-t} + \delta(t-1/2) \quad y(0) = 3 \quad y'(0) = -5$.

6. Find the Laplace transform of the π-periodic extension of the function $f(x) = x$ defined on the interval $[0, \pi]$.

7. Find all eigenfunctions and eigenvalues of the following Sturm-Liouville problem:
$$y'' + 2y' + (2+\lambda)y = 0, \quad y(0) = y(\pi) = 0.$$

8. Find sine, cosine and complex Fourier transforms of the function
$$f(x) = \begin{cases} 0, & x < 0 \\ xe^{-x}, & 0 \le x \end{cases}$$
Find also the Laplace transform of f. Try not to repeat calculations.

9. Let $f(x) = x$ be defined on the interval $[0, \pi]$.
   (1) Find the Fourier series of its π-periodic extension.
   (2) Find the sine and cosine series of f.
   (3) Draw the graphs of sums of all three series.
   (4) Substitute $x = 0$ into all three series. What are the sums of the resulting series? The same question for $x = -2$.

10. Suppose the Fourier series of some 2π-periodic function $f(x)$ starts from $1 + \sin x + \cos x$ (i.e. $a_0 = a_1 = b_1 = 1$). Could it happen that $|f(x)| \le 1$ for all x? (Hint: use Parseval's equality/Bessel's inequality).