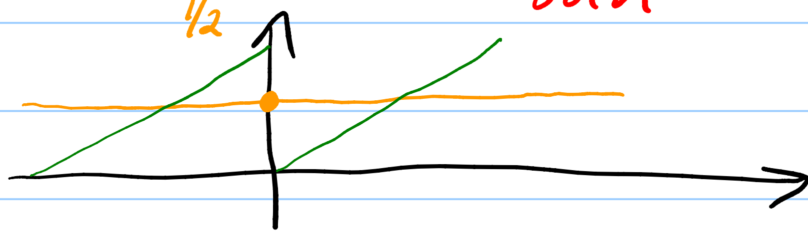
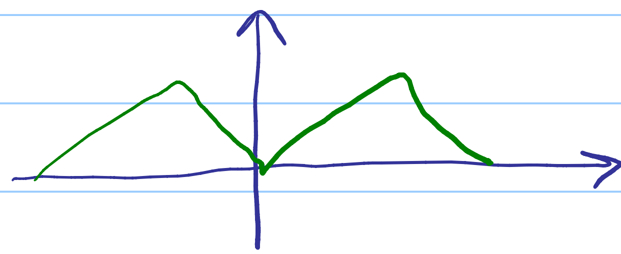
More solutions to practice problems 2

18. $f(x) = \frac{1}{2} + \underbrace{\sum (\text{Sine terms})}_{\text{odd}}$ ← odd fcn relative to line $y = \frac{1}{2}$

C. 

symmetric about $(0, 1/2)$

19. $\frac{1}{2} - \sum (\text{Cosine terms})$ ← even fcn

A. 

even

(B) is not even, not odd, not symm about $(0, 1/2)$.

(D) is odd, not symmetric about $(0, 1/2)$.

21. $$\underbrace{\tilde{\mathcal{F}}_c\Big[\underbrace{\tilde{\mathcal{F}}_c[e^{-x}]}_{=\sqrt{\frac{2}{\pi}}\,\frac{1}{1+w^2}}\Big]}_{} = e^{-x}$$

$$= \sqrt{\frac{2}{\pi}} \int_0^\infty \sqrt{\frac{2}{\pi}}\, \frac{1}{1+w^2} \cos wx \, dw = e^{-x}$$

So $$\frac{2}{\pi} \int_0^\infty \frac{\cos 2w}{1+w^2}\, dw = e^{-2}$$

23. $f(x) = $ Fourier Sine Series ← given

$$= \sum_{n=1}^{\infty} A_n \sin nx$$ ← all even terms $= 0$ except $A_2 = \frac{1}{2}$

Sol^n to string problem:

$$u(x,t) = \sum_{n=1}^{\infty} A_n \sin nx \cos nt$$ ← $c=1$

$$u\left(\tfrac{\pi}{4}, \tfrac{\pi}{2}\right) = \sum_{n=1}^{\infty} \underbrace{A_n}_{\substack{=0 \\ \text{for even} \\ n>2}} \sin \frac{n\pi}{4} \underbrace{\cos \frac{n\pi}{2}}_{=0 \text{ for odd } n}$$

$$= A_2 \sin \frac{\pi}{2} \cos \pi$$

$$= \left(\frac{1}{2}\right)(1)(-1) = -\frac{1}{2}$$

Note: The sol^n $u(x,t)$ is

$$u(x,t) = \frac{1}{2} \sin 2x \cos 2t + \frac{4}{\pi} \sum_{n=0}^{\infty} \frac{(-1)^n}{4-(2n+1)^2} \sin(2n+1)x \cos(2n+1)t$$

25. $$u(x,t) = \sum_{n=1}^{\infty} A_n \sin nx \, e^{-c^2 n^2 t}$$

$c=2$

A_n given, Fourier Sine Series for $f(x)$

$$= \frac{4}{\pi} \sum_{n=1}^{\infty} \frac{1-(-1)^n}{n^3} \sin nx \, e^{-4n^2 t}$$

27.

$$u(x,t) = \int_0^\infty \left(A(w)\cos wx + B(w)\sin wx\right)e^{-c^2w^2t}\,dw$$

$c=2$

$$A(w) = \frac{1}{\pi}\int_{-\infty}^{\infty} f(p)\cos wp\,dp = \frac{1}{\pi}\int_{-1}^{1}\cos wp\,dp$$

$$= \frac{2}{\pi}\int_0^1 \cos wp\,dp = \frac{2}{\pi}\left[\frac{1}{w}\sin wp\right]_0^1$$

$$= \frac{2}{\pi}\left(\frac{\sin w\cdot 1}{w} - \frac{\sin 0}{w}\right) = \frac{2}{\pi}\,\frac{\sin w}{w}$$

$$B(w) = \frac{1}{\pi}\int_{-1}^{1}\underbrace{\sin wp}_{\text{odd}}\,dp = 0$$

So $u(x,t) = \int_0^\infty \frac{2}{\pi}\frac{\sin w}{w}\cos wx\, e^{-4w^2t}\,dw$

28.

$$u(r,\theta) = a_0 + \sum_{n=1}^{\infty}\left(a_n\left(\frac{r}{R}\right)^n\cos n\theta + b_n\left(\frac{r}{R}\right)^n\sin n\theta\right)$$

a's, b's full Fourier Series coeff for $f(\theta)$.

But $f(\theta) = \cos 2\theta$ ← This is a Fourier Series! $a_2 = 1$ Rest $= 0$

So $u(r,\theta) = a_2\left(\frac{r}{1}\right)^2\cos 2\theta = 1\cdot r^2\cdot\cos 2\theta$

$R=1$

29. Sturm-Liouville Problem in X:

$$\underbrace{X'' - \lambda X = 0 \quad X'(0) = 0,\ X'(\pi) = 0}$$

e-fcns have zero derivatives at $0, \pi$

Cosine fcns! λ_n's negative, 0

T prob:

$T' = c^2 \lambda T$

$$\underbrace{T = e^{c^2 \lambda t}}$$

$$u(x,t) = \sum c_n \left(\begin{matrix} \text{e-fcns} \\ X_n \text{ for } \lambda_n \end{matrix} \right) e^{c^2 \lambda_n t}$$

Not hard to guess E.

Sol^n: $$u(x,t) = A_0 + \sum_{n=1}^{\infty} A_n \cos nx\ e^{-c^2 n^2 t}$$

n's match here

Fourier Cosine Series coeff for f given

$$= \frac{2}{\pi} - \frac{4}{\pi} \sum_{n=1}^{\infty} \frac{1}{4n^2 - 1} \cos(2n)x\ e^{-(2)^2 (2n)^2 t}$$