1. Let $\vec{F} = (3xy, -1, 1/(z + 1))$ be a force field.

(i) Compute the work done in displacement along the helix given by $\vec{r}(t) = (\sin t, \cos t, t)$ for $0 \leq t \leq \pi/2$.

(ii) Compute the work done by the same force $\vec{F}$ in displacement along the line from $(1,2,3)$ to $(2,3,3)$.
2. Let \( P = (0, 1, 0) \) be a point on a surface \( S \) given by \( y^4z = \ln(x + y) \).

(15) (i) Find the upward unit normal \( \vec{n} \) to \( S \) at \( P \).

Answer:

(10) (ii) Find the equation of the tangent plane to \( S \) at \( P \).

Answer:
3. Let \( \vec{F} = \langle x, y, z \rangle \) and \( T \) be the solid region given by \( x^2 + y^2 \leq z \leq 2 \).

(15) (i) Without using the divergence theorem, compute the surface integral \( \iint_S \vec{F} \cdot \vec{n} \, dA \), where \( S \) is the entire boundary of \( T \), and \( \vec{n} \) is the outward unit normal. Note that \( S \) is the sum of two smooth surfaces.

Answer:

(15) (ii) Using the divergence theorem, set up but do not evaluate, the volume integral corresponding to (i).

\[ \int \int \int \int dA \]
4. (15) Let $\vec{F} = \langle y, 0, 2 \rangle$, and $S$ be the surface given by $z = x + y$ for $0 \leq x \leq 1$ and $0 \leq y \leq 1$. Let $C$ be the curve bounding $S$ and traversed in the positive direction with respect to the upward unit normal $\vec{n}$ to $S$. Using Stokes’s theorem, compute the line integral $\int_C \vec{F} \cdot d\vec{r}$. (You do not have to compute the integral two ways; only by the area integral in Stokes’s theorem).

Answer: