Math 530

Midterm Exam

1. Suppose $f(z)$ is analytic in a neighborhood of a point a and has a simple zero at a, i.e., $f(a) = 0$, but $f'(a) \neq 0$. Prove a formula for the residue of $1/f(z)^2$ at $z = a$ involving values of derivatives of f at a. (Derive your formula without using any results from the practice problems.)

2. Compute

$$\int_{-\infty}^{\infty} \frac{e^{-ist}}{t^2 + 2t + 5}\, dt$$

if $s > 0$. Explain.

3. Suppose $f(z)$ has an isolated singularity at the origin and satisfies an estimate

$$|f(z)| \leq \frac{1}{\sqrt{|z|}} \quad \text{for } 0 < |z| < r$$

for some radius $r > 0$. Prove that the origin is a removable singularity of f.

4. Suppose f is an entire function that satisfies an estimate of the form

$$c|z|^N \leq |f(z)| \quad \text{if } |z| > R$$

for some positive integer N and positive real constants c and R. Prove that f must be a polynomial. What can you say about the degree of the polynomial?