

**Math 530**  
Midterm Exam

1. Suppose  $f(z)$  is analytic in a neighborhood of a point  $a$  and has a simple zero at  $a$ , i.e.,  $f(a) = 0$ , but  $f'(a) \neq 0$ . Prove a formula for the residue of  $1/f(z)^2$  at  $z = a$  involving values of derivatives of  $f$  at  $a$ . (Derive your formula without using any results from the practice problems.)

2. Compute

$$\int_{-\infty}^{\infty} \frac{e^{-ist}}{t^2 + 2t + 5} dt$$

if  $s > 0$ . Explain.

3. Suppose  $f(z)$  has an isolated singularity at the origin and satisfies an estimate

$$|f(z)| \leq \frac{1}{\sqrt{|z|}} \quad \text{for } 0 < |z| < r$$

for some radius  $r > 0$ . Prove that the origin is a removable singularity of  $f$ .

4. Suppose  $f$  is an entire function that satisfies an estimate of the form

$$c|z|^N \leq |f(z)| \quad \text{if } |z| > R$$

for some positive integer  $N$  and positive real constants  $c$  and  $R$ . Prove that  $f$  must be a polynomial. What can you say about the degree of the polynomial?