

Math 530

Quiz

1. Let $\text{Log } z$ denote the principal branch of the complex logarithm function. Compute the residue of

$$\frac{\text{Log } z}{(z^2 + 1)^2}$$

at the point $z = i$.

2. Prove carefully that

$$\int_{C_R} \frac{e^{iz}}{(z^2 + z)^2} dz$$

tends to zero as $R \rightarrow \infty$ where C_R is the upper semi-circle parameterized by $z(t) = Re^{it}$, $0 \leq t \leq \pi$.

3. Compute

$$\int_{C_2} \frac{z}{(z-1)(z+1)} dz$$

where C_2 denotes the circle of radius 2 about the origin parameterized in the standard sense. Show your work.

4. Write down a complex integral over the unit circle that represents the value of

$$\int_0^{2\pi} \frac{1}{\cos^4 t + \sin^4 t} dt.$$

DO NOT TRY TO COMPUTE THE INTEGRAL. And do not try to simplify it either.

5. What is the image of the unit disc under the map $\frac{z-1}{z+1}$? under the map $\frac{z-2}{z+2}$?