

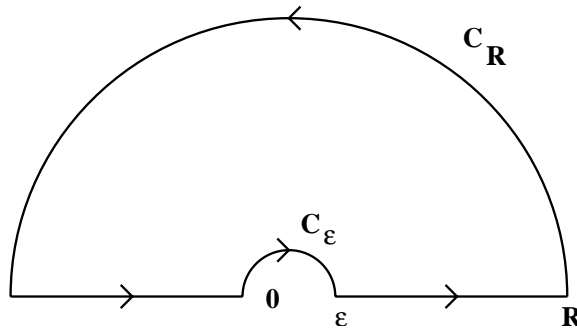
Math 530

Exam 2

1. Use the contour pictured below to compute

$$\int_0^{\infty} \frac{1}{\sqrt{x}(x^2 + 4)} dx.$$

Justify your calculations and limits.



2. Show that if f is an analytic mapping of the unit disc into itself such that $f(a) = 0$, then

$$|f(z)| \leq \left| \frac{z - a}{1 - \bar{a}z} \right|$$

for all z in the disc. What is the supremum of $|f'(a)|$ as f ranges over all such maps? Is the supremum attained by a map in the class?

3. a) Find an analytic function that maps $\{z : 0 < \operatorname{Re} z < 1\}$ one-to-one onto the first quadrant.
b) The linear fractional transformation $L(z) = \frac{z-1}{z+1}$ maps the domain $\mathbb{C} - [-1, 1]$ one-to-one onto a domain Ω in the complex plane. Determine Ω . Is Ω simply connected?

4. Prove that there are no monic polynomials of the form

$$P(z) = z^n + a_{n-1}z^{n-1} + \cdots + a_1z + a_0$$

satisfying $|P(z)| < 1$ when $|z| = 1$.

5. Prove the maximum principle for harmonic functions: If a harmonic function on a domain Ω assumes a local maximum, then it is constant on Ω .

Hint:

$$|e^{f(z)}| = e^{\operatorname{Re} f(z)}.$$