1. Find a one-to-one conformal mapping of the region common to the two disks 
\(|z - 1| < \sqrt{2}\) and \(|z + 1| < \sqrt{2}\) onto the unit disk.

2. Find a one-to-one conformal mapping of the region \(\{z : 0 < \text{Re } z < 1\}\) onto the unit disk.

3. Let \(\Omega\) denote the open set obtained by removing from \(\mathbb{C}\) the interval \([-1, 1]\). Prove that there is an analytic function \(F(z)\) on \(\Omega\) such that \(F(z)^2 = \frac{z+1}{z-1}\). Hint: What is the image of \(\Omega\) under the map \((z+1)/(z-1)\)?

4. Assume that \(f(z)\) is analytic and satisfies the inequality \(|f(z) - 1| < 1\) in a domain \(\Omega\). Prove that
\[
\int_{\gamma} \frac{f'(z)}{f(z)} \, dz = 0
\]
for every closed curve in \(\Omega\).

5. Suppose that \(f_n\) is a sequence of analytic functions on a domain \(\Omega\) which converges uniformly on compact subsets of \(\Omega\) to a non-constant function \(f\). Suppose that \(f\) has a zero of order \(m\) at a point \(a\) in \(\Omega\). Prove that there is an \(\epsilon > 0\) and a positive integer \(N\) such that each function \(f_n(z)\) with \(n > N\) has exactly \(m\) zeroes (counted with multiplicity) on \(D_{\epsilon}(a) \subset \Omega\).

6. Suppose that \(f_n\) is a sequence of analytic functions on a domain \(\Omega\) which converges uniformly on compact subsets of \(\Omega\) to a function \(f\). Suppose that \(\Omega\) is a domain containing \(f_n(\Omega)\) for each \(n\). Prove that, if \(f\) is not constant, then \(\Omega\) contains \(f(\Omega)\) too.

7. Compute
   a) \(\int_0^{\infty} \frac{x^{1/3}}{1 + x^2} \, dx\),  
   b) \(\int_0^{\infty} \frac{1}{1 + x^5} \, dx\),
   c) \(\int_{-\infty}^{\infty} \frac{x^2}{(x^2 + a^2)^3} \, dx\),  
   a real,  
   d) \(\int_{-\infty}^{\infty} \left(\frac{\sin x}{x}\right)^2 \, dx\).