RESEARCH PROPOSAL

Riemann Hypothesis

A proof is offered of the Riemann hypothesis not only for the classical Euler zeta function but also for its generalization using Dirichlet characters and for Hecke zeta functions constructed from modular forms.

The project is the result of special circumstances resulting from the second world war. At the age of nine the present investigator emigrated to the United States where he was brought up by his maternal grandparents. His grandfather, Dr. Ellice McDonald, was the director of the Biochemical Research Foundation, a privately funded organization of the Franklin Institute of Philadelphia. He was educated by his grandfather for a career of scientific research.

The Biochemical Research Foundation was founded by Irénée du Pont, the president of the du Pont Company who converted a manufacturer of munitions into a rival of the great German chemical companies. Mr. du Pont took a personal interest in the operation of the Biochemical Research Foundation. He also took a personal interest in the boy who was being trained as a future director of the foundation.

He did this by proposing a problem which originates from the solution of the Fermat problem given by Louis de Lagrange in the simplest nontrivial case. The problem is to find positive integers a, b, and c such that

$$a^3 + b^3 = 22c^3.$$

The present investigator found these numbers when he was fourteen. He was given an undergraduate education at the Massachusetts Institute of Technology, the same university as Mr. du Pont.

On graduation however he chose to go on to graduate school in mathematics, rather than chemistry as seemed indicated by the nature of biochemical research. His grandfather died as he was taking qualifying examinations at Cornell University. The remainder of his career was similar to that of other immigrants from Europe who arrived under less favorable circumstances.

The choice of mathematics as a career earned the lasting enmity of his uncle, Ellice McDonald Jr., who believed that this whimsical decision was a betrayal of his father's trust. My uncle did not himself have the talent for a research career.

The great majority of the public, whether in or out of mathematics, will endorse his uncle's position. This is the reason why the continuation by research on the Riemann hypothesis requires explanation

Research on the Riemann hypothesis is a continuation of the aims of the Biochemical Research Foundation. No other objectives would have survived the competition of government funding of research in biochemistry.

The underlying problem in the Riemann hypothesis concerns the quantum mechanical theory of orbital electrons in atoms. Quantum mechanics is formulated in Fourier analysis. The relevant quantum mechanics in formulated in Fourier analysis for the complex plane or for the four dimensional space of quaternions with real coordinates.

The discrete nature of quantum mechanics introduces related vector spaces of dimension two and four over the field of p–adic numbers for every prime p. Fourier analysis on these spaces modifies Fourier analysis on real vector spaces in a way which is new to quantum mechanics but not to mathematics. A new formulation of quantum mechanics is proposed in which there is a natural unit of distance, presumably the radius of a nucleus. An orbital electron acquires the symmetrics of a cube instead of just a sphere. These structures are possible in mathematics and therefore deserve attention as possibilities in nature.

It should be unnecessary to state that information concerning the properties of electrons in atoms is of the greatest importance in physics as well as chemistry.

The underlying concepts in the Riemann hypothesis are properties of Fourier analysis on a vector space of even dimension. A Radon transformation computes the Fourier transformation on the space from the Fourier transformation on a space of half the same dimension.

The proof of the Riemann hypothesis originates in the observation that the Radon transformation is maximal accretive. Accretive means that the sum of the transformation and its adjoint is nonnegative. Maximal means that the transformation cannot be extended to maintain the accretive property.

An underlying issue in the Riemann hypothesis is the determination of all functions which vanish in a given neighborhood of the origin and whose Fourier transform vanishes in the same neighborhood. A solution of this problem would greatly facilitate reception of the Riemann hypothesis theory.

The spectral theory of a vibrating string is another underlying issue in the Riemann hypothesis. The Riemann hypothesis constructs special strings whose properties are the focus of current research. They determine the forces which hold electrons in their orbits about a nucleus.

The theory of vibrating strings and its applications to Fourier analysis are formulated in the theory of certain Hilbert spaces of entire functions introduced by the present investigator during postdoctoral years. Methods of complex analysis are applied in an axiomatic treatment of special functions related to the gamma function. The zeta functions of the Riemann hypothesis succumb to this treatment.