HW # 11

This is due by 11:59pm, Sunday, April 21

1 Use the **Gram-Schmidt Process (GSP)** to find an *orthonormal basis* for $\mathbb{R}^3$, using this (nonstandard) basis for $\mathbb{R}^3$:

$$\mathcal{B} = \left\{ \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \right\}.$$

2 If $\mathbf{c}$ is a fixed vector in $\mathbb{R}^n$ and $W = \left\{ \mathbf{x} \in \mathbb{R}^n : \mathbf{x} \bullet \mathbf{c} = 0 \right\}$, show that W is a subspace of $\mathbb{R}^n$. (W is the set of all vectors in $\mathbb{R}^n$ orthogonal to $\mathbf{c}$.)

3 Find a basis for $W^\perp$, the *orthogonal complement* of W, when

$$W = \textbf{Span} \left\{ \begin{bmatrix} 2 \\ 1 \\ -1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 8 \\ 5 \\ -1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ -2 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 2 \\ 2 \\ 0 \end{bmatrix} \right\}.$$

4 Let $W = \textbf{Span} \left\{ \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix} \right\}$. Find the *projection* of the vector $\mathbf{v} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ onto the subspace W, i.e. compute $\textbf{proj}_W \mathbf{v}$.

5 Given the following data in the table below, use the **Least Squares Method** for a best

(a) **Linear fit**: $y = mx + b$. Use to predict the population at year 5.

(b) **Quadratic fit** : $y = ax^2 + bx + c$. Use to predict the population at year 5.

(For (b), you may use the computer to do some of the calculations since the numbers get big.)

Year x	Population (millions) y
1	1
2	3
3	7
4	6