

**Homework 1: MATH 490C / BIOL 595N, Due Wednesday, 1/18**

1. Find the fixed points, determine the stability of each fixed point, and sketch the phase portrait for each of the following equations.

(a)  $x' = 4x^2 - 16$

(b)  $x' = x - x^3$

(c)  $x' = -x - x^3$

(d)  $x' = x^2(4 - x^2)$

2. Consider the reaction  $A + B \xrightleftharpoons[k^-]{k^+} C$ .

- (a) Set up a system of differential equations to describe the change in concentration of each of  $A$ ,  $B$ , and  $C$ . You may assume that the rate of combination  $A + B$  is proportional to  $[A][B]$  and that there are no other reactions. Use  $a = [A]$ ,  $b = [B]$ ,  $c = [C]$ . Determine the equilibrium concentration of  $C$  as a function of the initial concentrations,  $a_0, b_0, c_0$ , the rate constants, and the equilibrium concentrations of  $A$  and  $B$ .
- (b) Determine the relationship between the concentrations  $a$  and  $c$ . Use this to express the equilibrium concentration of  $C$  in terms of the initial concentrations, the rate constants, and the equilibrium concentration of  $B$  (but not of  $A$ ).
- (c) Determine the relationship between  $a$  and  $b$ . Use this and the relationship between  $a$  and  $c$  to write a single differential equation of the form  $da/dt = f(a)$  that describes the behavior of this reaction.