Homework 5: MATH 490C / BIOL 595N, Due Wednesday, 2/15

THIS IS A GROUP HOMEWORK ASSIGNMENT: turn in one set of solutions per group and list the members of the group on your solution sheet.

1. For each of the following one-variable odes, use winpp/locbif to plot the bifurcation diagram using r as a parameter. Identify the type of each bifurcation, and be sure to plot all possible branches of the fixed points along with a label indicating stability or instability of each branch (locbif can be a little flaky, so use your brain as well as the computer!).

   (a) $\dot{x} = r - \cosh(x)$

   (b) $\dot{x} = x - rx(1-x)$

   (c) $\dot{x} = rx + 4x^3$

2. The Noble model for Purkinje cells in cardiac tissue has a form similar to the Morris-Lecar and Hodgkin-Huxley models in that it has voltage dependent gates controlling ionic currents. In this case there is an inward sodium current and an outward potassium current. The specific form of this model is

$$C_m \frac{dv}{dt} = -g_{\text{Na}}(V - V_{\text{Na}}) - (g_{\text{K}_1} + g_{\text{K}_2})(V - V_{\text{K}}),$$

where $C_m = 12$, $V_{\text{Na}} = 40$, and $V_{\text{K}} = -100$. The 2 different potassium conductances are due to the assumption that there are 2 types of potassium channel: one is an instantaneous, voltage-dependent channel while the other is time-dependent. The time-dependent channel has the form $g_{\text{K}_2} = 1.2n^4$, where n is a gating variable described below. The instantaneous channel has the form $g_{\text{K}_1} = 1.2\exp(-(V+90)/50) + .015\exp((V+90)/60)$. Finally, the sodium conductance has the form $g_{\text{Na}} = 400m^3h + g_i$, where $g_i = .14$ is a small inward sodium leak and m and h are gating variables described below.

The gating variables m, n, and h are described via

$$\dot{w} = \alpha_w(V)(1-w) - \beta_w(V)w,$$

where w may be any one of m, n, or h, and each of $\alpha_w(V)$ and $\beta_w(V)$ has the form

$$\sigma(V, V_0, C_1, C_2, C_3, C_4, C_5) = \frac{C_1\exp(C_2(V - V_0)) + C_3(V - V_0)}{1 + C_4\exp(\frac{V - V_0}{C_5})}, \qquad (1)$$

and the constants V_0 and C_1 through C_5 are given in the following table.

	C_1	C_2	C_3	C_4	C_5	V_0
α_m	0	1	.1	-1	-15	-48
β_m	0	1	$-.12$	-1	5	-8
α_h	.17	$-.05$	0	0	1	-90
β_h	1	0	0	1	-10	-42
α_n	0	1	.0001	-1	-10	-50
β_n	.002	$-.0125$	0	0	1	-90

   (a) Implement the Noble model in winpp and simulate for 2000ms (ms are the natural time units in the model). Start with all variables equal to 0 for initial conditions. Plot the voltage, which should show a train of action potentials (between 2 and 3 will show up in the first 2 seconds). Explain why the first action potential has a different shape from the others.

   Note: you can make your life easier by creating a function defined as σ in (1). Also, you can change the default integration time to 2000 by selecting Numerics → Int. Pars. and setting Time Stuff → Total to 2000.

   (b) Create plots like figure 2.7 (A) and (B) for each of g_{Na}, g_{K_1} and g_{K_2}. To do this, you will need to create a new ode file that contains the relavant part of the Noble model but which sets V as a parameter and sets the relevant current using the aux keyword. For plot (A), first find an equilbrium point at a fixed holding voltage of -60mv, then grab that point and integrate starting from there with voltage clamps ranging from -60mv to 20mv in increments of 20mv. Use a time scale appropriate for the current so that you can see clearly the rise to equilibrium to get a rough idea of the time scale (which is different for the different currents). For plot (B), use the equilibrium you started with for plot (A), then run locbif with V as the paramter along the x-axis and the appropriate current along the y-axis.