

Intersection Stable des Ensembles de Julia

Stably Intersecting Julia Sets of Polynomials

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Sommaire: Newhouse [1] a introduit la notion d'épaisseur et l'a utilisée démontrer que pour les ensembles de Cantor de la droite se coupent dans certaines conditions. Les définitions usuelles de l'épaisseur ne peuvent pas être généralisées à des dimensions supérieures, mais dans cet article nous construisons deux polynômes tels que toutes les fonctions holomorphes qui sont des C^2 -perturbations suffisamment petites de ces polynômes ont des ensembles de Julia qui contiennent des ensembles de Cantor qui se coupent. En outre, un de ces polynômes est de la forme $z^2 + c$ avec un grand $|c|$.

Abstract: Newhouse [1] introduced the concept of thickness and used it to prove that Cantor sets in the line must intersect under certain conditions. The usual definitions of thickness do not generalize to higher dimensions, but in this paper we produce two polynomials such that any holomorphic functions which are sufficiently small C^2 perturbations of these polynomials have Julia sets which contain intersecting Cantor sets. Moreover, one of the polynomials is of the form $z^2 + c$ with $|c|$ large.

1 Notation

Supposons que $D \subset \subset \mathbf{C}$ et que f_k est une application continue de \overline{D} dans D pour $k = 1, \dots, m$ et que ces applications ont des images disjointes. Soit J une suite d'indices $\{j_1, j_2, \dots, j_n\}$, soit $f_J := f_{j_1} \circ f_{j_2} \circ \dots \circ f_{j_n}$, $|J| := n$, et $jJ := \{j, j_1, \dots, j_n\}$. On définit

$$K_0(\{f_k\}) := \overline{D},$$
$$K_l(\{f_k\}) := \bigcup_j f_j(K_{l-1}(\{f_k\})) \text{ si } l \geq 1,$$

et $K(\{f_k\}) := \bigcap_l K_l(\{f_k\})$. Pour $\zeta_1, \zeta_2 \in T$, où T désigne le cercle unité, soit $\Delta(\zeta_1, \zeta_2; 0)$ la distance angulaire de ζ_1 à ζ_2 , c.-à.-d. $\Delta(\zeta_1, \zeta_2; 0) := \arccos(1 - |\zeta_1 - \zeta_2|^2/2) \in [0, \pi]$. Pour $z \neq 0$, soit $\text{sign}(z) := z/|z|$, et pour $w_0 \in \mathbf{C}$ et $z, w \in \mathbf{C} - \{w_0\}$, soit $\Delta(z, w; w_0) := \Delta(\text{sign}(z - w_0), \text{sign}(w - w_0); 0)$. Si $f(z) := az + b$, alors $\Delta(f(z), f(w); f(w_0)) = \Delta(z, w; w_0)$. Pour $\zeta \in T$, $z \in \mathbf{C}$, $0 < r_1 < r_2$, et $\delta_\theta \in [0, \pi]$, on définit

$$W_\zeta(z; r_1, r_2, \delta_\theta) := \{w : |w - z| \in [r_1, r_2] \text{ et } \Delta(w, z + \zeta; z) \leq \delta_\theta\}.$$

Cet ensemble est l'intersection de la couronne circulaire ayant pour centre z et rayons r_1 et r_2 avec le cône ayant pour centre z et angle au sommet $2\delta_\theta$, lequel est bissecté par le segment de z à $z + \zeta$.

2 Une variante linéaire d'intersection

Nous donnons ici les idées du principal lemme d'intersection dans le cas où toutes les applications qui définissent les ensembles de Cantor sont linéaires. Supposons que D_1 est un disque dans \mathbf{C} et que $g_1, g_2: \overline{D_1} \rightarrow D_1$ sont de la forme $g_k(z) = a_k z + b_k$ avec des images disjointes, $0 < |a_k| < 1$, et que $\text{sign}(a_1)$ n'est pas une racine de l'unité. Alors g_1 a un point fixe unique z_1 , et il y a un $K \in \mathbf{Z}_+$ tel que pour tout $\zeta \in T$ il y a $0 \leq j \leq K$ avec $g_1^j g_2(z_1) \in W_\zeta(z_1; r_1, r_2, \pi/8)$, où $r_1 = |z_1 - g_1^K g_2(z_1)|$ et $r_2 = |z_1 - g_2(z_1)|$. Soit $S := \{g_1^j g_2(z_1) : 0 \leq j \leq K\}$. Remarquons que S est un sous-ensemble de $K(\{g_k\})$, de même que $g_J(S)$ pour tout J .

Nous produisons des applications $\{f_k\}$ telles que $K(\{g_k\})$ coupent $K_l(\{f_k\})$ pour chaque l d'une manière stable. Soit $K_l := K_l(\{f_k\})$. Choisissons N tel que $1/N < |a_1|^K |a_2|$ et soit D_2 un carré qui contient z_1 ayant les côtés de longueur $4r_2 N$. Définissons N^2 applications f_j de la forme $f_j(z) = q_j + z(1 - \delta)/N$, où les q_j 's sont choisis tel que les images de f_j 's sont des sous-ensembles disjointes de D_2 pour tout $\delta > 0$. Choisissons δ assez petit pour que pour n'importe quel z dans $\overline{D_2}$, il y a un $\zeta \in T$, avec $W_\zeta(z; |a_1| r_1, r_2, \pi/8) \subset K_1$. Puisque $z_1 \in D_2$, il y a un $w_1 \in S \cap K_1$, supposons que $w_1 = g_1^{j_1} g_2(z_1)$, avec $w_1 \in f_{k_1}(\overline{D_2})$. En utilisant la linéarité de f_k 's et en écrivant $r = (1 - \delta)/N$, il y a un $\zeta_1 \in T$ tel que $W_{\zeta_1}(w_1; r|a_1| r_1, r r_2, \pi/8)$ est contenue dans $f_{k_1}(K_1)$, et donc dans K_2 .

Remarquons que $g_1^{j_1} g_2 g_1^j(S)$ coupent $W_\zeta(w_1; R r_1, R r_2, \pi/8)$ pour tout $\zeta \in T$, où $R = |a_1|^{j_1+j} |a_2|$, $j \geq 0$. Puisque $r < |a_1|^K |a_2|$ et $j_1 \leq K$, nous pouvons choisir $j \geq 0$ tel que $r|a_1| \leq R \leq r$, et dans ce cas $W_{\zeta_1}(w_1; R r_1, R r_2, \pi/8) \subseteq W_{\zeta_1}(w_1; r|a_1| r_1, r r_2, \pi/8)$. Cela donne un point w_2 contenu dans $g_1^{j_1} g_2 g_1^j(S) \cap K_2$, et ce processus peut être continué inductivement pour montrer que les $K(\{g_k\})$ coupent chaque K_l , et donc les deux ensembles Cantor se coupent. Un changement suffisamment petit dans a_j, b_j, q_j , ou δ ne détruira pas cette intersection.

3 Contrôle de la nonlinéarité et la intersection des ensembles de Julia

Si les applications g_j et f_j sont remplacées par des perturbations holomorphes, l'argument ci-dessus peut être appliqué avec des modifications mineures en employant (1), (2), (3). Nous avons donc obtenu la proposition suivante.

Proposition 1 *Soit $D_1 \subset\subset \mathbf{C}$ un convexe et soient G_1 et G_2 des applications bi-holomorphes dans un voisinage de $\overline{D_1}$ avec $G_j(\overline{D_1}) \subseteq D_1$ et $G_1(\overline{D_1}) \cap G_2(\overline{D_1}) = \emptyset$. Supposons que $C_1 < 1$ tel que $|G_j(z) - G_j(w)| < C_1 |z - w|$ pour $z, w \in \overline{D_1}$. Soit $a_1 = z_1$ le point fixe de G_1 , et $\delta_\theta \in (0, \pi/8)$, et supposons que $\text{sign}(G_1'(z_1))$ n'est pas une racine d'ordre m de l'unité pour $m \leq 3\pi/\delta_\theta$. Il y a alors une région $D_2 \subset\subset \mathbf{C}$ et des applications biholomorphes $\{F_j\}_{j=1}^n: \overline{D_2} \rightarrow D_2$ avec des images disjointes tels que si g_j et f_j sont des C^2 -perturbations holomorphes suffisamment petites de G_j et F_j respectivement, alors $K(\{g_j\}) \cap K(\{f_j\}) \neq \emptyset$.*

Construisons des polynômes dont les ensembles de Julia contiennent des ensembles de Cantor qui se coupent stablement, soit $|c| > 8$. Soit D le disque de centre 0 et rayon $|c|/2$. Définissons G_j sur \overline{D} , par $G_1(z) := \sqrt{z - c}$, $G_2(z) := -\sqrt{z - c}$,

pour un certain choix de la branche de la racine carré. Pour un sous-ensemble dense et ouvert de $\{|c| > 8\}$, on emploie la Proposition 1 pour obtenir des applications $\{F_j\}_{j=1}^n$ tels que si g_j et f_j sont des C^2 -perturbations holomorphes suffisamment petites de G_j et F_j , alors $K(\{g_j\}) \cap K(\{f_j\}) \neq \emptyset$. En approximant la collection $\{F_j^{-1}\}$ avec un polynôme P tel que P a n branches inverses différentes P_j^{-1} qui sont C^2 -près de F_j , on déduit que des fonctions holomorphes G et F qui sont C^2 -près de $z^2 + c$ et P auront des inverses qui sont C^2 -près de G_j et F_j , et par conséquent les ensembles de Julia de G et F se couperont.

1 Notation

Suppose that $D \subset\subset \mathbf{C}$ and that f_k is a continuous map of \overline{D} into D for $k = 1, \dots, m$ and that these maps have disjoint images. Let J stand for a sequence of indices $\{j_1, j_2, \dots, j_n\}$ and write f_J for $f_{j_1} \circ f_{j_2} \circ \dots \circ f_{j_n}$. Also, let $|J| = n$, and $jJ = \{j, j_1, \dots, j_n\}$. Define

$$\begin{aligned} K_0(\{f_k\}) &:= \overline{D}, \\ K_l(\{f_k\}) &:= \bigcup_j f_j(K_{l-1}(\{f_k\})) \text{ if } l \geq 1, \end{aligned}$$

and put $K(\{f_k\}) := \bigcap_l K_l(\{f_k\})$. For $\zeta_1, \zeta_2 \in T$, the unit circle, let $\Delta(\zeta_1, \zeta_2; 0)$ be the angular distance from ζ_1 to ζ_2 ; i.e., $\Delta(\zeta_1, \zeta_2; 0) := \arccos(1 - |\zeta_1 - \zeta_2|^2/2) \in [0, \pi]$. For $z \neq 0$ define $\text{sign}(z) := z/|z|$, and for $w_0 \in \mathbf{C}$ and $z, w \in \mathbf{C} - \{w_0\}$ set $\Delta(z, w; w_0) := \Delta(\text{sign}(z - w_0), \text{sign}(w - w_0); 0)$. If $f(z) := az + b$, then $\Delta(f(z), f(w); f(w_0)) = \Delta(z, w; w_0)$. Finally, for $\zeta \in T$, $z \in \mathbf{C}$, $0 < r_1 < r_2$, and $\delta_\theta \in [0, \pi]$, define

$$W_\zeta(z; r_1, r_2, \delta_\theta) := \{w : |w - z| \in [r_1, r_2] \text{ and } \Delta(w, z + \zeta; z) \leq \delta_\theta\}.$$

This set is the intersection of the annulus having center z and radii r_1 and r_2 with the cone having vertex z and angle of opening $2\delta_\theta$ which is bisected by the segment from z to $z + \zeta$.

2 A linear version of intersection

We sketch here the ideas of the main intersection lemma when all of the maps defining the Cantor sets are linear. Suppose that D_1 is a disk in \mathbf{C} and that $g_1, g_2: \overline{D}_1 \rightarrow D_1$ are of the form $g_k(z) = a_k z + b_k$ with disjoint images, $0 < |a_k| < 1$, and $\text{sign}(a_1)$ not a root of unity. Then g_1 has a unique fixed point z_1 and there exists $K \in \mathbf{Z}_+$ so that given $\zeta \in T$ there exists $0 \leq j \leq K$ with $g_1^j g_2(z_1) \in W_\zeta(z_1; r_1, r_2, \pi/8)$, where $r_1 = |z_1 - g_1^K g_2(z_1)|$ and $r_2 = |z_1 - g_2(z_1)|$. Set $S := \{g_1^j g_2(z_1) : 0 \leq j \leq K\}$, and note that S is a subset of $K(\{g_k\})$, as is $g_J(S)$ for any J .

We next produce maps $\{f_k\}$ so that $K(\{g_k\})$ intersects $K_l(\{f_k\})$ for each l in some stable manner. Write K_l for $K_l(\{f_k\})$. Choose N so that $1/N < |a_1|^K |a_2|$ and let D_2 be a square containing z_1 with sides of length $4r_2 N$. Define N^2 maps f_j of the form $f_j(z) = q_j + z(1 - \delta)/N$ where the q_j 's are chosen so that the images of

the f_j 's are disjoint subsets of D_2 for any $\delta > 0$. Choose δ small enough that given any z in $\overline{D_2}$, there exists $\zeta \in T$ with $W_\zeta(z; |a_1|r_1, r_2, \pi/8) \subset\subset K_1$. Since $z_1 \in D_2$, there exists some $w_1 \in S \cap K_1$, say $w_1 = g_1^{j_1} g_2(z_1)$ with $w_1 \in f_{k_1}(\overline{D_2})$. Using the linearity of the f_k 's and writing $r = (1 - \delta)/N$, there exists some $\zeta_1 \in T$ so that $W_{\zeta_1}(w_1; r|a_1|r_1, rr_2, \pi/8)$ is contained in $f_{k_1}(K_1)$, and hence in K_2 .

Note that $g_1^{j_1} g_2 g_1^j(S)$ intersects $W_\zeta(w_1; Rr_1, Rr_2, \pi/8)$ for any $\zeta \in T$, where $R = |a_1|^{j_1+j}|a_2|$, $j \geq 0$. Since $r < |a_1|^K|a_2|$ and $j_1 \leq K$, we may choose $j \geq 0$ so that $r|a_1| \leq R \leq r$, in which case $W_{\zeta_1}(w_1; Rr_1, Rr_2, \pi/8) \subseteq W_{\zeta_1}(w_1; r|a_1|r_1, rr_2, \pi/8)$. This gives a point w_2 contained in $g_1^{j_1} g_2 g_1^j(S) \cap K_2$, and this process can be continued inductively to show that $K(\{g_k\})$ intersects each K_l and hence that the two Cantor sets intersect. Any sufficiently small change in a_j, b_j, q_j , or δ will not destroy this intersection.

3 Control of nonlinearity

With the basic approach to intersection as outlined above, we next consider the behavior of certain holomorphic maps in order to apply the above proof to more general situations. We start with some lemmas.

Lemma 1 *Suppose $D \subset\subset \mathbf{C}$ and that $f_j: \overline{D} \rightarrow D$ is holomorphic for $j = 1, \dots, m$, and suppose there exist constants $C_1 < 1$ and $C_2, C_3 > 0$ such that for all j and for all $z, w \in \overline{D}$ we have $C_2|z - w| < |f_j(z) - f_j(w)| < C_1|z - w|$ and $|f_j(z) - f_j(w) - (z - w)f_j'(w)| < C_3|z - w|^2$. For a sequence $J = \{j_1, \dots, j_n\}$ and $w \in \overline{D}$, let $T_J^w(z) := f_J(w) + (z - w)(f_J)'(w)$. Then for any J and any $z, w \in \overline{D}$ we have*

$$\frac{|f_J(z) - f_J(w)|}{|T_J^w(z) - T_J^w(w)|} \leq \prod_{j=0}^{|J|-1} \left(1 - \frac{C_3 C_1^j}{C_2} |z - w| \right) \leq \frac{|f_J(z) - f_J(w)|}{|T_J^w(z) - T_J^w(w)|}, \quad (1)$$

$$\frac{|(f_J)'(z)|}{|(f_J)'(w)|} \leq \prod_{j=0}^{|J|-1} \left(\frac{2C_3 C_1^j}{C_2} |z - w| + 1 \right), \quad (2)$$

and if $0 < C_3|z - w| < C_2/\sqrt{2}$, then

$$\Delta(f_J(z), T_J^w(z); f_J(w)) \leq \sum_{j=0}^{|J|-1} \frac{\sqrt{2} C_3 C_1^j}{C_2} |z - w|. \quad (3)$$

The proof of this lemma is by induction on $|J|$ for each inequality.

The next lemma shows that for certain holomorphic contractions defined on a region D and having disjoint images, we can still construct a sequence of points like that obtained from $K(\{g_k\})$ in the linear version of intersection.

Lemma 2 *Let $D \subset\subset \mathbf{C}$ be convex and let G_1 and G_2 be biholomorphic on a neighborhood of \overline{D} with $G_j(\overline{D}) \subseteq D$ and $G_1(\overline{D}) \cap G_2(\overline{D}) = \emptyset$. Suppose $C_1 < 1$ such that $|G_j(z) - G_j(w)| < C_1|z - w|$ for $z, w \in \overline{D}$. Let $a_1 = z_1$ be the fixed point of G_1 , let $\delta_\theta \in (0, \pi/4)$, and suppose that $\text{sign}(G_1'(z_1))$ is not an m th root of unity*

for $m \leq 3\pi/\delta_\theta$. Then there exist $r, s, \delta_s > 0$ and points $\{a_j\} \subseteq K(\{G_j\})$ defined inductively as follows. For $k \geq 1$, given $r_k \in (0, r)$ and $\zeta_k \in \partial\Delta$, we can choose $a_{k+1} \in W_{\zeta_k}(a_k; \delta_s s \prod_{j=2}^k r_j, s \prod_{j=2}^k r_j, \delta_\theta)$. This construction can be carried out with the same r, s , and δ_s for all small enough holomorphic C^2 perturbations of G_1 and G_2 .

Proof: The assumptions on G_j imply the existence of constants $C_2, C_3 > 0$ as in lemma 1. Let $0 < \epsilon < \min\{\frac{1}{3}, \frac{1}{C_1} - 1\}$. Choose $\delta > 0$ so that $C_3\delta < C_2/2$, $\exp(\frac{4C_3\delta}{C_2(1-C_1)}) < 1 + \epsilon$ and $\frac{2\sqrt{2}C_3\delta}{C_2(1-C_1)} < \epsilon\delta_\theta$. An application of lemma 1 shows that if $w_1, w_2, w \in \overline{D}$ with $0 < |w_j - w| < \delta$, then

$$\frac{|G_J(w_1) - G_J(w)|}{|G_J(w_2) - G_J(w)|} \in \left(\left(\frac{1}{1+\epsilon} \right) \frac{|w_1 - w|}{|w_2 - w|}, (1+\epsilon) \frac{|w_1 - w|}{|w_2 - w|} \right) \quad (4)$$

and

$$\Delta(G_J(w_1), G_J(w_2); G_J(w)) < \Delta(w_1, w_2; w) + \epsilon\delta_\theta. \quad (5)$$

To construct the desired sequence of points, choose $K \geq 0$ so that $|z_1 - G_1^K G_2(z_1)| < \delta$. Let $d := G_1^K G_2(z_1)$ and choose s with $\max\{C_1, 3/4\}|z_1 - d| < s < |z_1 - d|$. The assumption on $\text{sign}(G_1'(z_1))$ gives $k_0 \in \mathbb{Z}_+$ such that for any $\zeta \in T$ we can find $k \in \{1, \dots, k_0\}$ with $\Delta(\zeta, (G_1'(z_1))^k; 0) < \delta_\theta/3$. Finally, choose $\delta_s \in (0, \delta)$ and $r > 0$ so that

$$|G_1^{k_0+1}(d) - z_1| > 2\delta_s s \quad \text{and} \quad r < \delta_s s C_2 / ((1+\epsilon)|z_1 - G_2(z_1)|). \quad (6)$$

Given $\zeta_1 \in T$, we have $\Delta(\zeta_1, (d - z_1)(G_1'(z_1))^k; 0) < \delta_\theta/3$ for some $1 \leq k \leq k_0$. Using linear invariance for Δ with (3) and the choice of δ shows that $\Delta(z_1 + \zeta_1, G_1^k(d); z_1) < 2\delta_\theta/3$. Also $|G_1^k(d) - z_1| < s$ and $|G_1^k(d) - z_1| \geq |G_1^{k_0+1}(d) - z_1| > 2\delta_s s$, so taking $a_2 = G_1^k(d)$ gives $a_2 \in W_{\zeta_1}(a_1; \delta_s s, s, \delta_\theta)$, as desired.

To induct, let $\zeta_m \in T$ and $r_m \in (0, r)$, and suppose that $a_{m-1} = G_J(z_1)$ and $a_m = G_J G_1^l(d)$ for some J and $l \geq 1$. Then $a_m = G_J G_1^{l+K} G_2(z_1)$, and we will choose a_{m+1} of the form $G_J G_1^{l+K} G_2 G_1^k(d)$ for some $k \geq 1$. Since $a_{m-1} = G_J G_1^{l+K}(z_1)$, we can use (4) together with a lower bound on $|a_{m-1} - a_m|$ and the choice of r to find $k \geq 1$ with

$$|G_{J'} G_1^{k-1}(d) - a_m| > s \prod_{j=2}^m r_j \quad \text{and} \quad |G_{J'} G_1^k(d) - a_m| \leq s \prod_{j=2}^m r_j, \quad (7)$$

where $G_{J'} = G_J G_1^{l+K} G_2$. Writing $a_m = G_{J'} G_1^{k-1}(z_1)$, we can use (4) together with (6) to conclude for $1 \leq j \leq k_0$ that

$$\frac{|G_{J'} G_1^{k+j}(d) - a_m|}{|G_{J'} G_1^{k-1}(d) - a_m|} \geq \left(\frac{1}{1+\epsilon} \right) \frac{|G_1^{j+1}(d) - z_1|}{|d - z_1|} \geq \delta_s.$$

Thus for $1 \leq j \leq k_0$, (7) gives $|G_{J'} G_1^{k+j}(d) - a_m| > \delta_s s \prod_{j=2}^m r_j$, and a similar calculation gives an upper bound of $s(\prod_{j=2}^m r_j)$ for $1 \leq j \leq k_0$. From this and (5) we conclude the existence of $j \in \{1, \dots, k_0\}$ so that taking $a_{m+1} = G_{J'} G_1^{k+j}(d)$ completes the induction.

If g_j is holomorphic on \overline{D} with $\|G_j^{(k)} - g_j^{(k)}\|_{L^\infty(\overline{D})}$ sufficiently small for $k = 0, 1, 2$, then we may choose the same $\delta, \epsilon, s, \delta_s$, and r and proceed as above.

The next lemma gives a nonlinear version of the maps used in section 2.

Lemma 3 *Given $s > 0$, $r, \delta_s \in (0, 1)$, and $\delta_\theta \in (0, \pi/8)$, there exist a region $D \subset \subset \mathbf{C}$ and biholomorphic maps $\{F_j\}_{j=1}^n : \overline{D} \rightarrow D$ such that if f_j is holomorphic and a sufficiently small C^2 perturbation of F_j for each j , then $f_j(\overline{D}) \cap f_k(\overline{D}) = \emptyset$ if $j \neq k$, $\|f_j'\|_{L^\infty(\overline{D})} < r$, and given $z \in f_J(\overline{D})$, there exists $\zeta \in T$ and j so that for $z = f_J(w)$ and $R = |(f_J)'(w)|$ we have*

$$W_\zeta(z; R\delta_s s, Rs, \delta_\theta) \subseteq f_J f_j(\overline{D}).$$

The proof of this lemma consists of taking the F_j 's to be linear maps as in section 2, then applying the results of lemma 1 together with the argument principle.

4 Intersection

We are finally in a position to produce stably intersecting Cantor sets. The following proposition is a direct generalization of the linear intersection result to the case of nonlinear maps.

Proposition 1 *Suppose the region D_1 and maps G_1, G_2 satisfy the hypotheses of lemma 2 for some $\delta_\theta \in (0, \pi/8)$. Then there exist a region $D_2 \subset \subset \mathbf{C}$ and biholomorphic maps $\{F_j\}_{j=1}^n : \overline{D}_2 \rightarrow D_2$ with disjoint images such that if g_j and f_j are holomorphic and sufficiently small C^2 perturbations of G_j and F_j respectively, then $K(\{g_j\}) \cap K(\{f_j\}) \neq \emptyset$.*

Proof: Apply lemma 3 and lemma 2 in alternation using (2) from lemma 1.

To construct polynomials whose Julia sets contain stably intersecting Cantor sets, let $|c| > 8$, let D be the disk centered at 0 with radius $|c|/2$, and define G_j on \overline{D} by $G_1(z) := \sqrt{z-c}$, $G_2(z) := -\sqrt{z-c}$, for some branch of square root. For an open dense subset of $\{|c| > 8\}$, proposition 1 applies to give maps $\{F_j\}_{j=1}^n$ such that if g_j and f_j are small enough holomorphic C^2 perturbations of G_j and F_j , then $K(\{g_j\}) \cap K(\{f_j\}) \neq \emptyset$. Approximate the collection $\{F_j^{-1}\}$ with a polynomial P so that P has n distinct inverse branches P_j^{-1} which are C^2 close to F_j . Then any holomorphic functions G and F which are C^2 close to $z^2 + c$ and P will have inverses which are C^2 close to G_j and F_j , and hence the Julia sets of G and F will intersect.

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