SAMPLE PROBLEMS FOR MIDTERM II MATH 265

All vectors in this list are represented in row vectors for the convenience of typewriting.

(1) Find a vector v in R^2 that has twice the magnitude of $u_1 = (-3, 4)$ and is in the same direction as $u_2 = (1, 2)$.

(2) The matrix

$$\begin{bmatrix} 1 & 0 & 2 & 0 & 1 \\ 0 & 1 & 2 & 0 & -1 \\ 0 & 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

is a reduced row echelon form of the matrix

$$A = \begin{bmatrix} 1 & 1 & 4 & 1 & 2 \\ 0 & 1 & 2 & 1 & 1 \\ 1 & -1 & 0 & 0 & 2 \\ 2 & 1 & 6 & 0 & 1 \end{bmatrix}$$

(a) Find the nullity of A and a basis of nullspace of A.
(b) Find the rank of A
(c) Find the basis of columns space
(d) Find the basis of row space
(e) Find a basis of the orhogonal complement of row space of A.

(3) Express, if possible, one of the vectors of the following set as a linear combination of the others: $v_1 = (1, -1, 0)$, $v_2 = (0, 2, 1)$, $v_3(1, -2, 3)$, $v_4 = (3, 6, 2)$.

(4) For what values of d is the vector $(2, 1, d)$ in the span of $(1, 2, 1)$ and $(2, 5, 1)$.

(5) Let $S = \{v_1, v_2, v_3\}$, where $v_1 := (1, 2, 2)$, $v_2 := (3, 2, 1)$ $v_3 = (7, 6, 4)$. Determine if S is lineraly independent. Find a basis for the subspace $W = spanS$ of R^3 and its dimension . Determine whether $u = (1, 6, 5)$ is in W.

(6) For the problem (a) (b) (c) determine whether the given set of vectors is linearly inedpendent or linearly dependent:
(a) $\{(1, 0, -1), (2, 1, 1)\}$
(b) $\{(1, 0, -1), (2, 1, -1), (0, 1, 3)\}$
(c) $\{(1, 0, -1), (1, 3, 4), (2, 1, -1), (0, 1, 3)\}$

For the problem (d) (e) (f) determine whether the given set of vectors spans R^3
(d) $\{(1, 1, -1), (2, 1, 1)\}$
(e) $\{(1, 0, 0), (1, 3, 4), (2, 1, 0), (0, 1, 3)\}$
(f) $\{(1, 0, 0), (1, 3, 4), (2, 1, 0)\}$

For the problem (g) (h) (i) determine whether the given set of vectors form a basis of R^3
(g)$(1, 2, 0)$, $(1, 1, 1)$, $(1, 0, 1)$.
(h)$(1, 2, 0)$, $(1, 1, 1)$, $(2, 3, 1)$
(i)$(1, 2, 0)$, $(1, 1, 1)$

(7) Let $W = \{(1, -1, -1, 2), (0, 1, 2, 0), (2, -1, 0, 2), (1, 0, 1, 2)\}$ be a set of vectors in R^4. Find a basis of subspace of $spanW$ consisting of some vectors in W.

(8) Let W be the subspace of R^4 consisting of all vectors of the form $(b + c, a - b, 2b - a + c, a + c)$. Find a basis of W and its dimension.

(9) Find a basis of $\{(x, y, z) \mid x + y + 2z = 0\}$.

(10) Find a basis of R^4 that includes $(1, 1, 1, 0)$, $(0, 0, 1, 1)$.

(11) Let $S = \{v_1, v_2, v_3\}$ and $T = \{w_1, w_2, w_3\}$, be two bases of R^3, where $v_1 := (1, 3, 2)$, $v_2 := (1, 1, 1)$, $v_3 := (1, 2, 2)$ and $w_1 := (0, 3, 1)$, $w_2 := (1, 1, 1)$ $w_3 := (0, 1, 2)$, . Compute the coordinates of $[v]_T$ and $[v]_S$ of the vector $v = (0, 2, 4)$.

(12) Find an orthogonal basis of of $\{(x, y, z) \mid x + y + 2z = 0\}$

(13) Find an orthonormal basis for the subspace of R^4 with basis $(1, 1, 0, 1), (1, 0, 0, -1), (1, 0, 1, -1)$.

(14) Let W be the subspace of R^4 spanned by $\{(1, 1, 1, 0), (-1, 0, 1, -1)$. Find an orthonormal basis for the subspace $W^{\perp}$

(15) Find the orthonormal basis of R^4 wich contains $(1/\sqrt{2}, 0, 0, 1/\sqrt{2})$ and $(0, 1/\sqrt{2}, 1/\sqrt{2}, 0)$

(16) Let W be the subspace of R^3 with orthogonal basis $\{(1, 0, 1), (2, 1, -2)\}$. Write a vector u as $u = v + w$ where $v \in W$ and $w \in W^{\perp}$

(17) Let W be the plane of R^3 with equation $x + y + 2z = 0$. Write a vector u as $u = v + w$ where $v \in W$ and $w \in W^{\perp}$

(18) Find the projection $Proj_W(v)$ of the vector $(2, 2, 6)$ onto the subspace spanned by $v_1 = (-1, 0, 1)$ and $v_2 = (1, 2, 1)$.

(19) Find the distance from the point $(1, 2, 3)$ to the plane $x + y + z = 0$

(20) Let W be the null space of the matrix

$$B = \begin{bmatrix} 1 & 2 & 0 \\ 2 & 0 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

Find the basis of $W^{\perp}$.

(21) Find the least square fit line for the points $(-1, 3), (0, 2), (1, 4)$

(22) Let $L : R_3 \to R_3$ be a linear transformation for which we know that $L([1, 0, 1]) = [1, 2, 3]$, $L([0, 1, 2]) = [1, 0, 0]$ and $L([1, 1, 0]) = [1, 0, 1]$.
(a) What is $L([4, 1, 0])$?
(b) What is $L([0, 0, 0])$?
(c) What is $L([a, b, c])$?
(d) Find the standard matrix that represents the given linear transformation L.