**MA 271: Several Variable Calculus**

**EXAM I (practice)**

NAME ____________________________ Lecture Meeting Time ____________________________

**NO CALCULATORS, BOOKS, OR PAPERS ARE ALLOWED. Use the back of the test pages for scrap paper.**

**Points awarded**

**1. (5 pts)** ____________ **7. (5 pts)** ____________

**2. (5 pts)** ____________ **8. (5 pts)** ____________

**3. (5 pts)** ____________ **9. (5 pts)** ____________

**4. (5 pts)** ____________ **10. (5 pts)** ____________

**5. (5 pts)** ____________ **11. (5 pts)** ____________

**6. (5 pts)** ____________ **12. (5 pts)** ____________

**Total Points:** ____________

**1. Find the normal vector to the plane $3x + 2y + 6z = 6$**

**A. (0, 0, 1)**
**B. (-3, -2, -6)**
**C. (1, 1, 1)**
**D. (0, 0, 1)**
**E. (1/3, 1/2, 1/6)**

**2. Find distance from point $(1, 1, 3)$ to the plane $3x + 2y + 6z = 6$**

**A. 17/7**
**B. 3**
**C. 12/5**
**D. 0**
**E. 11/7**

**3. The surface defined by $y^2 - x^2 = z$ is a**

**A. hyperbolic paraboloid**
**B. elliptical cone**
**C. elliptical paraboloid**
**D. ellipsoid**
**E. hyperboloid**

**4. Find the speed of the particle with position function $\vec{r}(t) = e^{3t}\,\mathbf{i} + e^{-3t}\,\mathbf{j} + te^{3t}\,\mathbf{k}$ when $t = 0$.**

**A.** $\mathbf{i} + 2\,\mathbf{j} + \mathbf{k}$

**B.** $1$

**C.** $\sqrt{2}$

**D.** $\sqrt{17}$

**E.** $\sqrt{19}$

**5. The plane $S$ passes through the point $P(1, 2, 3)$ and contains the line $x = 3t$, $y = 1 + t$, and $z = 2 - t$. Which of the following vectors is normal to $S$?**

**A.** $\mathbf{i} + 2\,\mathbf{j} + \mathbf{k}$

**B.** $\mathbf{i} - 2\,\mathbf{j} + \mathbf{k}$

**C.** $\mathbf{i} + \mathbf{k}$

**D.** $\mathbf{i} - 2\,\mathbf{j}$

**E.** $\mathbf{i} + 2\,\mathbf{j}$

**6. Which of the following statements is true for all three-dimensional vectors $\vec{a}, \vec{b}$, and $\vec{c}$, if $\theta$ is the angle between $\vec{a}$ and $\vec{b}$?**

**(i)** $\vec{a} \times \vec{b} = \vec{b} \times \vec{a}$

**(ii)** $\vec{a} \cdot (\vec{b} \times \vec{c}) = (\vec{b} \times \vec{c}) \cdot \vec{a}$

**(iii)** $|\vec{a} \times \vec{b}| = |\vec{a}| \cdot |\vec{b}| \cdot |\cos\theta|$

**(iv)** $(\vec{a} \times \vec{b}) \cdot a = 0$

**A. All are true**

**B. (i) and (ii) only**

**C. (i), (ii), and (iv) only**

**D. (iii) and (iv) only**

**E. (ii) and (iv) only**

**7. A particle starts at the origin with initial velocity $\vec{i}+\vec{j}-\vec{k}$. Its acceleration is $\vec{a}(t)=t\vec{i}+\vec{j}+t\vec{k}$. Find its position at $t=1$.**

**A.** $\frac{1}{6}\ \vec{i}+\frac{1}{2}\ \vec{j}+\frac{1}{3}\ \vec{k}$

**B.** $\frac{7}{6}\ \vec{i}+\frac{1}{2}\ \vec{j}-\frac{5}{6}\ \vec{k}$

**C.** $\vec{i}+\vec{j}+\vec{k}$

**D.** $\frac{7}{6}\ \vec{i}+\frac{3}{2}\ \vec{j}-\frac{5}{6}\ \vec{k}$

**E.** $\vec{i}+2\vec{j}-\vec{k}$

**8. Find the arc length of the curve defined by $\vec{r}(t)=(t,\frac{\sqrt{6}}{2}t^2,t^3),-1\leq t\leq 1$.**

**A. 5**

**B. 4**

**C. 3**

**D. 2**

**E. 6**

**9. Find the equation of the plane that contains the points $(1,2,1)$, $(2,-1,0)$ and $(3,3,1)$.**

**A.** $-x-2y+9z=4$

**B.** $x+2y+7z=12$

**C.** $x-2y+7z=4$

**D.** $x+2y+z=6$

**E.** $-x+2y+9z=12$

**10. Find parametric equations for the tangent line to the curve**

$$\vec{r}(t) = (t^2 + 3t + 2, e^t \cos t, \ln(t+1))$$

**at** $t = 0$**.**

**A.** $x = 2 + 3t \quad y = 1 + t \quad z = t$

**B.** $x = 2t + 3, \; y = e^t(\cos t - \sin t), \; z = \dfrac{1}{t+1}$

**C.** $x = 3 + 2t \quad y = 1 + t \quad z = 1$

**D.** $x = 3t \quad y = 2t \quad z = 1 + t$

**E.** $x = 2 - t \quad y = 1 + t \quad z = 3 - 3t$

**11. Let** $C$ **be the intersection of** $x^2 + y^2 = 16$ **and** $x + y + z = 5$**. Find the curvature at** $(0, 4, 1)$**.**

**A.** $\frac{1}{8}\sqrt{\frac{3}{2}}$

**B.** $5$

**C.** $6$

**D.** $\frac{2}{3}$

**E.** $\sqrt{9/10}$

**12. The plane** $S$ **passes through the point** $P(1, 2, 3)$ **and contains the line** $x = 3t$**,** $y = 1 + t$**, and** $z = 2 - t$**. Which of the following is an equation for** $S$**?**

**A.** $x + 2y + z = 0$

**B.** $x - 2y + z = 0$

**C.** $x - 2y + z = 5$

**D.** $x + 2y + z = 5$

**E.** $x - y + z = 5$

**13. Find the unit tangent vector T of $\vec{r}(t) = (\sin(3t))\vec{i} + (\cos(3t))\vec{j} + (4t)\vec{k}$ at any $t$.**

**A.** $\mathbf{T} = \frac{3}{5}\cos(3t)\vec{i} - \frac{3}{5}\sin(3t)\vec{j} + \frac{4}{5}\vec{k}$

**B.** $\mathbf{T} = \frac{3}{5}\sin(3t)\vec{i} - \frac{3}{5}\cos(3t)\vec{j} + \frac{4}{5}\vec{k}$

**C.** $\mathbf{T} = 3\cos(3t)\vec{i} - 3\sin(3t)\vec{j} + 4\vec{k}$

**D.** $\mathbf{T} = \sin(3t)\vec{i} - \cos(3t)\vec{j} + 4t\vec{k}$

**E.** $\mathbf{T} = 1$

**14. Find the unit normal vector N of $\vec{r}(t) = (\sin(3t))\vec{i} + (\cos(3t))\vec{j} + (4t)\vec{k}$ at any $t$.**

**A.** $\mathbf{N} = \frac{3}{5}\cos(3t)\vec{i} - \frac{3}{5}\sin(3t)\vec{j} + \frac{4}{5}\vec{k}$

**B.** $\mathbf{N} = \frac{3}{5}\sin(3t)\vec{i} - \frac{3}{5}\cos(3t)\vec{j} + \frac{4}{5}\vec{k}$

**C.** $\mathbf{N} = 3\cos(3t)\vec{i} - 3\sin(3t)\vec{j} + 4\vec{k}$

**D.** $\mathbf{N} = -\sin(3t)\vec{i} + \cos(3t)\vec{j}$

**E.** $\mathbf{N} = 1$

**15. Find the torsion of the curve $\vec{r}(t) = (\sin(3t))\vec{i} + (\cos(3t))e^t\vec{j}$ at any $t = 3$.**

**A.** $\cos(9)e^3$

**B.** $e^3$

**C.** 3

**D.** 0

**E.** 1

**16. Find the equation for the surface consisting of all points $P$ for which the distance to the $x$–axis is twice the distance from $P$ to the $yz$–plane. Identify the surface.**

**17. Find an equation of the plane that passes through the point $P(-1, 2, 1)$ and contains the line of intersection of the planes $x + y - z = 2$ and $2x - y + 3z = 1$.**

**18. (a) Find the point of intersection of the lines $x = 2t + 1,\ y = 3t + 2,\ z = 4t + 3$ and $x = s + 2,\ y = 2s + 4,\ z = -4s - 1$.**

**(b) Find the plane determined by these lines.**

**19. Let $C$ be the intersection of $x^2 + y^2 = 16$ and $x + y + z = 5$. Find a parametric equation for $C$.**

**20. Find the unit tangent vector T, the principle unit normal vector N and the unit binormal vector B of $\mathbf{r}(t) = (3\sin(t))\mathbf{i} + (3\cos(t)\mathbf{j} + 4t\mathbf{k}$ at any $t$.**
**Recall: $\mathbf{N} = \frac{\frac{d\mathbf{T}}{dt}}{|\frac{d\mathbf{T}}{dt}|}$ and $\mathbf{B} = \mathbf{T} \times \mathbf{N}$.**

**21. Calculate the tangential and normal components of the acceleration for $\vec{r}(t) = 2t\vec{i} + t^2\vec{j} + \frac{1}{3}t^3\vec{k}$.**
**Recall $a = a_T\mathbf{T} + a_N\mathbf{N}$ and $a_T = \frac{d}{dt}|v|$**

**22.**

$$\lim_{n\to\infty} (\sin(\frac{2}{n}))^{1/n} =$$

**A. 1**

**B.** 0

**C.** 2

**D.** $e$

**E. diverge**

**23.**

$$\lim_{n\to\infty} n\sin(\frac{2}{n}) =$$

**A.** 1

**B.** 0

**C.** 2

**D.** $e$

**E. diverge.**

**24. What is the sum of**

$$\sum_{n=1}^{\infty} \sin(n\pi)$$

**A.** 1

**B.** 0

**C.** 2

**D.** $\frac{1}{2}$

**E. diverge**

**25. What is the sum of**

$$\sum_{n=1}^{\infty} \sin(\frac{n\pi}{2})$$

**A.** 1

**B.** 0

**C.** 2

**D.** $\frac{1}{2}$

**E. diverge**

**26. What is the value of $m$ if**

$$\sum_{n=1}^{\infty} \frac{1}{2^{n+5}} = \sum_{n=m}^{\infty} \frac{1}{2^n} =$$

**A.** 1

**B.** 0

**C.** 5

**D.** $-5$

**E.** 6

**27. What is the value $b$ such that**

$$1 + e^b + e^{2b} + e^{3b} + \cdot = 9$$

**A.** $-\ln(\frac{9}{8})$

**B.** $\ln(\frac{9}{8})$

**C.** $\frac{8}{9}$

**D.** $-2$

**E.** $-1$

**28. Make up an infinite series of nonzero terms whose sum is $5$.**

**29. Make up two infinite convergent geometric series**

$$\sum_{n=1}^{\infty} a_n = A, \quad \sum_{n=1}^{\infty} b_n = B \textbf{ and } \sum_{n=1}^{\infty} a_n * b_n \neq AB$$

**30. Find the values of $x$, such that the following series converge**

$$\sum \frac{(x-2)^n}{10^n}.$$

**31. Determine whether the given series is convergent or divergent. In each case, name the test you used: n-th term test (divergence test), integral test, comparison test, limit comparison test, alternating series test, ratio test, root test and show your work. Namely, you are required to answer the following 4 questions.**
**Q1: converge or diverge;**
**Q2: which test is used;**
**Q3: state the test;**
**Q4: how it is used.**

**A.** $\sum_{n=1}^{\infty} \frac{-5^{n-1}}{4^n}$

**B.** $\sum_{n=1}^{\infty} \frac{n}{n^2+1}$

**C.** $\sum_{n=2}^{\infty} \frac{1}{n(\ln(n))^2}$

**D.** $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{1+\sqrt{n}}$

**E.** $\sum_{n=1}^{\infty} \frac{(-n)^{n+1}}{1+3n}$

**F.** $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{\sqrt{n+1}}{\sqrt{n}+1}$

**G.** $\sum_{n=1}^{\infty} \frac{n}{2^n}$

**H.** $\sum_{n=2}^{\infty} \frac{n}{(\ln(n))^n}$

**I.** $\sum_{n=1}^{\infty} (\frac{1}{1+n})^n$