

MA 271: Several Variable Calculus

EXAM I (practice)

NAME _____ Lecture Meeting Time _____

NO CALCULATORS, BOOKS, OR PAPERS ARE ALLOWED. Use the back of the test pages for scrap paper.

Points awarded

- | | |
|------------------|-------------------|
| 1. (5 pts) _____ | 7. (5 pts) _____ |
| 2. (5 pts) _____ | 8. (5 pts) _____ |
| 3. (5 pts) _____ | 9. (5 pts) _____ |
| 4. (5 pts) _____ | 10. (5 pts) _____ |
| 5. (5 pts) _____ | 11. (5 pts) _____ |
| 6. (5 pts) _____ | 12. (5 pts) _____ |

Total Points: _____

1. Find the normal vector to the plane $3x + 2y + 6z = 6$

- A. $(0, 0, 1)$
- B. $(-3, -2, -6)$
- C. $(1, 1, 1)$
- D. $(0, 0, 1)$
- E. $(1/3, 1/2, 1/6)$

2. Find distance from point $(1, 1, 3)$ to the plane $3x + 2y + 6z = 6$

- A. $17/7$
- B. 3
- C. $12/5$
- D. 0
- E. $11/7$

3. The surface defined by $y^2 - x^2 = z$ is a

- A. hyperbolic paraboloid
- B. elliptical cone
- C. elliptical paraboloid
- D. ellipsoid
- E. hyperboloid

4. Find the speed of the particle with position function $\vec{r}(t) = e^{3t} \mathbf{i} + e^{-3t} \mathbf{j} + te^{3t} \mathbf{k}$ when $t = 0$.

A. $\mathbf{i} + 2\mathbf{j} + \mathbf{k}$

B. 1

C. $\sqrt{2}$

D. $\sqrt{17}$

E. $\sqrt{19}$

5. The plane S passes through the point $P(1, 2, 3)$ and contains the line $x = 3t$, $y = 1 + t$, and $z = 2 - t$. Which of the following vectors is normal to S ?

A. $\mathbf{i} + 2\mathbf{j} + \mathbf{k}$

B. $\mathbf{i} - 2\mathbf{j} + \mathbf{k}$

C. $\mathbf{i} + \mathbf{k}$

D. $\mathbf{i} - 2\mathbf{j}$

E. $\mathbf{i} + 2\mathbf{j}$

6. Which of the following statements is true for all three-dimensional vectors \vec{a} , \vec{b} , and \vec{c} , if θ is the angle between \vec{a} and \vec{b} ?

(i) $\vec{a} \times \vec{b} = \vec{b} \times \vec{a}$

(ii) $\vec{a} \cdot (\vec{b} \times \vec{c}) = (\vec{b} \times \vec{c}) \cdot \vec{a}$

(iii) $|\vec{a} \times \vec{b}| = |\vec{a}| \cdot |\vec{b}| \cdot |\cos \theta|$

(iv) $(\vec{a} \times \vec{b}) \cdot \vec{a} = 0$

A. All are true

B. (i) and (ii) only

C. (i), (ii), and (iv) only

D. (iii) and (iv) only

E. (ii) and (iv) only

7. A particle starts at the origin with initial velocity $\vec{i} + \vec{j} - \vec{k}$. Its acceleration is $\vec{a}(t) = t\vec{i} + \vec{j} + t\vec{k}$. Find its position at $t = 1$.

A. $\frac{1}{6}\vec{i} + \frac{1}{2}\vec{j} + \frac{1}{3}\vec{k}$

B. $\frac{7}{6}\vec{i} + \frac{1}{2}\vec{j} - \frac{5}{6}\vec{k}$

C. $\vec{i} + \vec{j} + \vec{k}$

D. $\frac{7}{6}\vec{i} + \frac{3}{2}\vec{j} - \frac{5}{6}\vec{k}$

E. $\vec{i} + 2\vec{j} - \vec{k}$

8. Find the arc length of the curve defined by $\vec{r}(t) = (t, \frac{\sqrt{6}}{2}t^2, t^3)$, $-1 \leq t \leq 1$.

A. 5

B. $\boxed{4}$

C. 3

D. 2

E. 6

9. Find the equation of the plane that contains the points $(1, 2, 1)$, $(2, -1, 0)$ and $(3, 3, 1)$.

A. $-x - 2y + 9z = 4$

B. $x + 2y + 7z = 12$

C. $\boxed{x - 2y + 7z = 4}$

D. $x + 2y + z = 6$

E. $-x + 2y + 9z = 12$

10. Find parametric equations for the tangent line to the curve

$$\vec{r}(t) = (t^2 + 3t + 2, e^t \cos t, \ln(t + 1))$$

at $t = 0$.

A. $x = 2 + 3t \quad y = 1 + t \quad z = t$

B. $x = 2t + 3, \quad y = e^t(\cos t - \sin t), \quad z = \frac{1}{t + 1}$

C. $x = 3 + 2t \quad y = 1 + t \quad z = 1$

D. $x = 3t \quad y = 2t \quad z = 1 + t$

E. $x = 2 - t \quad y = 1 + t \quad z = 3 - 3t$

11. Let C be the intersection of $x^2 + y^2 = 16$ and $x + y + z = 5$. Find the curvature at $(0, 4, 1)$.

A. $\frac{1}{8}\sqrt{\frac{3}{2}}$

B. 5

C. 6

D. $\frac{2}{3}$

E. $\sqrt{9/10}$

12. The plane S passes through the point $P(1, 2, 3)$ and contains the line $x = 3t$, $y = 1 + t$, and $z = 2 - t$. Which of the following is an equation for S ?

A. $x + 2y + z = 0$

B. $x - 2y + z = 0$

C. $x - 2y + z = 5$

D. $x + 2y + z = 5$

E. $x - y + z = 5$

13. Find the unit tangent vector \mathbf{T} of $\vec{r}(t) = (\sin(3t))\vec{i} + (\cos(3t))\vec{j} + (4t)\vec{k}$ at any t .

A. $\mathbf{T} = \frac{3}{5}\cos(3t)\vec{i} - \frac{3}{5}\sin(3t)\vec{j} + \frac{4}{5}\vec{k}$

B. $\mathbf{T} = \frac{3}{5}\sin(3t)\vec{i} - \frac{3}{5}\cos(3t)\vec{j} + \frac{4}{5}\vec{k}$

C. $\mathbf{T} = 3\cos(3t)\vec{i} - 3\sin(3t)\vec{j} + 4\vec{k}$

D. $\mathbf{T} = \sin(3t)\vec{i} - \cos(3t)\vec{j} + 4t\vec{k}$

E. $\mathbf{T} = 1$

14. Find the unit normal vector \mathbf{N} of $\vec{r}(t) = (\sin(3t))\vec{i} + (\cos(3t))\vec{j} + (4t)\vec{k}$ at any t .

A. $\mathbf{N} = \frac{3}{5}\cos(3t)\vec{i} - \frac{3}{5}\sin(3t)\vec{j} + \frac{4}{5}\vec{k}$

B. $\mathbf{N} = \frac{3}{5}\sin(3t)\vec{i} - \frac{3}{5}\cos(3t)\vec{j} + \frac{4}{5}\vec{k}$

C. $\mathbf{N} = 3\cos(3t)\vec{i} - 3\sin(3t)\vec{j} + 4\vec{k}$

D. $\mathbf{N} = -\sin(3t)\vec{i} + \cos(3t)\vec{j}$

E. $\mathbf{N} = 1$

15. Find the torsion of the curve $\vec{r}(t) = (\sin(3t))\vec{i} + (\cos(3t))e^t\vec{j}$ at any $t = 3$.

A. $\cos(9)e^3$

B. e^3

C. 3

D. $\boxed{0}$

E. 1

16. Find the equation for the surface consisting of all points P for which the distance to the x -axis is twice the distance from P to the yz -plane. Identify the surface.

Answer: $y^2 + z^2 = 4x^2$, elliptical cone

17. Find an equation of the plane that passes through the point $P(-1, 2, 1)$ and contains the line of intersection of the planes $x + y - z = 2$ and $2x - y + 3z = 1$.

Answer: $x - 2y + 4z = -1$

18. (a) Find the point of intersection of the lines $x = 2t + 1$, $y = 3t + 2$, $z = 4t + 3$ and $x = s + 2$, $y = 2s + 4$, $z = -4s - 1$.

(b) Find the plane determined by these lines.

Answer: (a) $x = (1, 2, 3)$, (b) $20x - 12y - z = -7$

19. Let C be the intersection of $x^2 + y^2 = 16$ and $x + y + z = 5$. Find a parametric equation for C .

Answer: $x = 4 \cos(t), y = 4 \sin(t), z = 5 - 4 \cos(t) - 4 \sin(t)$

20. Find the unit tangent vector \mathbf{T} , the principle unit normal vector \mathbf{N} and the unit binormal vector \mathbf{B} of $\mathbf{r}(t) = (3 \sin(t))\mathbf{i} + (3 \cos(t))\mathbf{j} + 4t\mathbf{k}$ at any t .

Recall: $\mathbf{N} = \frac{\frac{d\mathbf{T}}{dt}}{\left|\frac{d\mathbf{T}}{dt}\right|}$ and $\mathbf{B} = \mathbf{T} \times \mathbf{N}$.

Answer: $\mathbf{T} = \left(\frac{3}{5} \cos(t), -\frac{3}{5} \sin(t), \frac{4}{5}\right)$

Answer: $\mathbf{N} = (-\sin(t), -\cos(t), 0)$

Answer: $\mathbf{B} = \left(\frac{4}{5} \cos(t), -\frac{4}{5} \sin(t), -\frac{3}{5}\right)$

21. Calculate the tangential and normal components of the acceleration for $\vec{r}(t) = 2t\vec{i} + t^2\vec{j} + \frac{1}{3}t^3\vec{k}$.

Recall $a = a_T\mathbf{T} + a_N\mathbf{N}$ and $a_T = \frac{d}{dt}|v|$

Answer: $a_T = 2t, a_N = 2$

22.

$$\lim_{n \rightarrow \infty} \left(\sin \left(\frac{2}{n} \right) \right)^{1/n} =$$

- A. 1
- B. 0
- C. 2
- D. e
- E. diverge

23.

$$\lim_{n \rightarrow \infty} n \sin \left(\frac{2}{n} \right) =$$

- A. 1
- B. 0
- C. 2
- D. e
- E. diverge.

24. What is the sum of

$$\sum_{n=1}^{\infty} \sin(n\pi)$$

- A. 1
- B. 0
- C. 2
- D. $\frac{1}{2}$
- E. diverge

25. What is the sum of

$$\sum_{n=1}^{\infty} \sin\left(\frac{n\pi}{2}\right)$$

- A. 1
- B. 0
- C. 2
- D. $\frac{1}{2}$
- E. diverge

26. What is the value of m if

$$\sum_{n=1}^{\infty} \frac{1}{2^{n+5}} = \sum_{n=m}^{\infty} \frac{1}{2^n} =$$

- A. 1
- B. 0
- C. 5
- D. -5
- E. 6

27. What is the value b such that

$$1 + e^b + e^{2b} + e^{3b} + \dots = 9$$

- A. $-\ln\left(\frac{9}{8}\right)$
- B. $\ln\left(\frac{9}{8}\right)$
- C. $\frac{8}{9}$
- D. -2
- E. -1

28. Make up an infinite series of nonzero terms whose sum is 5.

Answer: one possible choice is $\sum_{n=1}^{\infty} \frac{5}{2^n}$

29. Make up two infinite convergent geometric series

$$\sum_{n=1}^{\infty} a_n = A, \quad \sum_{n=1}^{\infty} b_n = B \quad \text{and} \quad \sum_{n=1}^{\infty} a_n * b_n \neq AB$$

Answer: one possible answer is $a_n = b_n = \frac{1}{2^n}$, where $\sum_{n=1}^{\infty} a_n = \sum_{n=1}^{\infty} b_n = 1$ but $\sum_{n=1}^{\infty} a_n * b_n = \frac{1}{3} \neq AB$ which is 1.

30. Find the values of x , such that the following series converge

$$\sum \frac{(x-2)^n}{10^n}.$$

Answer: $-8 < x < 12$

31. Determine whether the given series is convergent or divergent. In each case, name the test you used: n-th term test (divergence test), integral test, comparison test, limit comparison test, alternating series test, ratio test, root test and show your work. Namely, you are required to answer the following 4 questions.

Q1: converge or diverge;

Q2: which test is used;

Q3: state the test;

Q4: how it is used.

A. $\sum_{n=1}^{\infty} \frac{-5^{n-1}}{4^n}$

B. $\sum_{n=1}^{\infty} \frac{n}{n^2+1}$

C. $\sum_{n=2}^{\infty} \frac{1}{n(\ln(n))^2}$

D. $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{1+\sqrt{n}}$

E. $\sum_{n=1}^{\infty} \frac{(-n)^{n+1}}{1+3n}$

F. $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{\sqrt{n+1}}{\sqrt{n+1}}$

G. $\sum_{n=1}^{\infty} \frac{n}{2^n}$

H. $\sum_{n=2}^{\infty} \frac{n}{(\ln(n))^n}$

I. $\sum_{n=1}^{\infty} \left(\frac{1}{1+n}\right)^n$

Answer:

A. diverge, nth-term test

B. diverge, limit comparison test

C. converge, integral test

D. converge, alternating series test

E. converge, alternating series test

F. diverge, nth-term test

G. converge, ratio test

H. converge, root test

I. converge, root test