MA 271: Several Variable Calculus

EXAM II (practice)

NAME ____________________ Lecture Time ____________________

NO CALCULATORS, BOOKS, OR PAPERS ARE ALLOWED. Use the back of the test pages for scrap paper.

Points awarded

1. (5 pts) ________	**7. (5 pts)** ________
2. (5 pts) ________	**8. (5 pts)** ________
3. (5 pts) ________	**9. (5 pts)** ________
4. (5 pts) ________	**10. (5 pts)** ________
5. (5 pts) ________	**11. (5 pts)** ________
6. (5 pts) ________	**12. (5 pts)** ________

Total Points: ________

1. Find the radius of convergence of the power series

$$\sum_{n=0}^{\infty}(-3)^n\sqrt{n+1}(x+1)^{2n+1}$$

Answer: $\frac{1}{\sqrt{3}}$

2. Find the second degree Taylor polynomial of $f(x) = \dfrac{1-\cos(2x)}{3x^2}$ **with center** $x_0 = 0$**.**

Answer: $\frac{2}{3} - \frac{2}{9}x^2$

3. Find the second degree Taylor polynomial of $f(x) = \sqrt{x}$ **with center** $x_0 = 4$**.**
Answer: $2 + \frac{1}{4}(x-4) - \frac{1}{32\sqrt{2}}(x-4)^2$

4. If $L = \lim_{(x,y,z)\to(0,3,4)} \dfrac{x + 5y - 5z}{\sqrt{x^2 + y^2 + z^2}}$**, then**

A. $L = -3$

B. $L = -2$

C. $\boxed{L = -1}$

D. $L = 0$

E. the limit does not exist

5. If $L = \lim_{(x,y,z)\to(0,0,0)} \dfrac{x + 2y - 3z}{\sqrt{x^2 + y^2 + z^2}}$**, then**

A. $L = 1$

B. $L = -2$

C. $L = -3$

D. $L = 0$

E. $\boxed{\textbf{the limit does not exist}}$

6. If $f(x, y) = \ln(x + 2y^2)$**, then the partial derivative** f_{xy} **equals**

A. $\dfrac{-2x}{(x + 2y^2)^2}$

B. $\boxed{\dfrac{-4y}{(x + 2y^2)^2}}$

C. $\dfrac{4xy}{(x + 2y^2)^2}$

D. $\dfrac{-8xy}{(x + 2y^2)^2}$

E. $\dfrac{4(x^2 - y^2)}{(x + 2y^2)^2}$

7. Find $\dfrac{\partial z}{\partial y}$ at $(-2, 2, 2)$ if $z(x, y)$ is defined by the equation

$$xe^y + ye^z = 0$$

A. -1

B. $-\frac{1}{2}$

C. 0

D. $\boxed{\frac{1}{2}}$

E. 1

8. Find $\dfrac{\partial z}{\partial y}$ at $(1, \ln 2, \ln 3)$ if $z(x, y)$ is defined by the equation

$$xe^y + ye^z + 2\ln x - 2 - 3\ln 2 = 0.$$

A. $2 + \ln 2$

B. $\frac{4}{3\ln 2}$

C. $-\frac{5}{3\ln 2+1}$

D. $\boxed{-\frac{5}{3\ln 2}}$

E. 1

9. Let

$$f(x,y) = \begin{cases} \frac{y^2}{x^2+y^2}, & (x,y) \neq (0,0) \\ 0, & (x,y) = (0,0) \end{cases}$$

Find $f_x(0,0)$ and $f_y(0,0)$.

Answer: $f_x(0,0) = 0$ **and** $f_y(0,0)$ **does not exist.**

10. Suppose $z = f(x, y)$, where $x = e^t$ and $y = t^2 + 3t + 2$. Given that $\dfrac{\partial z}{\partial x} = x - y$ and $\dfrac{\partial z}{\partial y} = -x$, find $\dfrac{dz}{dt}$ when $t = 0$.

A. -6

B. $\boxed{-4}$

C. 6

D. 9

E. 15

11. Let $\mathbf{F} = f(x,y,z)\,\mathbf{i} + g(x,y,z)\,\mathbf{j} + h(x,y,z)\,\mathbf{k}$ and $\nabla = \partial_x\,\mathbf{i} + \partial_y\,\mathbf{j} + \partial_z\,\mathbf{k}$, which of the followings are NOT defined?

I. $\nabla \mathbf{F}$

II. $\nabla \cdot \mathbf{F}$

III. $\nabla \times \mathbf{F}$

IV. $\nabla(\nabla \cdot \mathbf{F})$

V. $\nabla \times (\nabla \times \mathbf{F})$

A. $\boxed{\textbf{I only}}$

B. **II and III**

C. **IV and V**

D. **I, II and IV**

E. **III and V**

12. Find the directional derivative of the function $f(x, y, z) = x^2y^2z^6$ at the point $(1, 1, 1)$ in the direction of the vector $\langle 2, 1, -2\rangle$.

A. -6

B. $\boxed{-2}$

C. 0

D. 2

E. 6

13. Find the direction in which the function $z = x^2 + 3xy - \frac{1}{2}\, y^2$ is increasing most rapidly at $(-1, -1)$.

A. $3i$

B. $5\vec{i} + 2\vec{j} - \vec{k}$

C. $\boxed{-5\vec{i} - 2\vec{j}}$

D. $2\vec{i} - 5\vec{j}$

E. $\sqrt{29}$

14. Consider the function $f(x, y) = 2x^2 - 3xy + y^2$. Find two unit vectors such that the directional derivative of f at the point $(1, 1)$ in these two directions is 1.

Answer: $(1, 0)$ **and** $(0, -1)$

15. By using a linear approximation of $f(x, y) = \sqrt{x^2 + y}$ at $(4, 9)$, compute the approximate value of $f(5, 8)$.

A. 5.2

B. 5.3

C. 5.5

D. $\boxed{5.7}$

E. 5.9

16. **The volume of a right circular cone with base radius r and height h is $V = \frac{\pi}{3}r^2h$. Suppose the radius is measured to be $6m \pm .2m$, and the height is measured to be $12m \pm .3m$. The volume calculated use differentials is $a \pm b\ m^3$. What are the values of a and b?**

Answer: $a = 144\pi, b = 13.2\pi$

17. **Let**

$$S\colon x = u - v,\ \ y = uv,\ \ z = u + v^2$$

If $(0, b, 5)$ is a point on the tangent plane to S at $(0, 1, 2)$ on S, then $b =$

A. $\boxed{3}$

B. 1

C. -2

D. 0

E. 2

18. **Find a equation for the tangent plane of**

$$\cos(\pi x) - x^2y + e^{xz} + yz = 4 \quad \textbf{at} \quad (0, 1, 2)$$

Answer: $2x + 2y + z - 4 = 0$

19. Find a parametric equation for the line passing through $P = (5, 2, 0)$, and normal to the tangent plane of

$$y^2 + z^2 = 4$$

at P.

A. $x = 0, y = t, z = 0$

B. $x = 5, y = 4t, z = 3t$

C. $x = 5t, y = 2t, z = 3t$

D. $\boxed{x = 5, y = 4t + 2, z = 0}$

E. $x = 5t + 5, y = 2t + 2, z = 3t$

20. For the function $f(x, y) = x^3 + 2y^2 + xy - 2x + 5y$, the point $(-1, -1)$ yields

A. a local minimum

B. a local maximum

C. $\boxed{\textbf{a saddle point}}$

D. $\nabla f(-1, -1) \neq 0$

E. The Second Derivative Test gives no information at $(-1, -1)$

21. Find absolute maximum and minimum values of

$$f(x, y) = x^2 + 2y^2 - x$$

on the disc $x^2 + y^2 \leq 1$.

Answer: max=$\frac{9}{4}$, min=$-\frac{1}{4}$

22. **The function** $f(x, y) = y\sin(x)$ **has**

A. **infinitely many local maximum points.**

B. **infinitely many local minimum points.**

C. $\boxed{\text{infinitely many saddle points.}}$

D. **exactly one local minimum point and one maximum point.**

E. **no critical point.**

23. **The max and min values of** $f(x, y, z) = xyz$ **on the surface** $2x^2 + 2y^2 + z^2 = 2$ **are**

A. $\pm \frac{\sqrt{2}}{9}$

B. $\pm \frac{\sqrt{3}}{9}$

C. $\boxed{\pm \frac{\sqrt{6}}{9}}$

D. $\pm \frac{2\sqrt{2}}{9}$

E. $\pm \frac{2\sqrt{3}}{9}$

24. **If we use the method of Lagrange multipliers to find the maximum of** $f(x, y) = 2x^2 - y^2 - y$ **subject to the constraint** $x^2 + y^2 = 1$**, the Lagrange multipliers** λ **that we find are:**

A. $\boxed{\lambda = 2}$

B. $\lambda = 0$

C. $\lambda = -1$

D. $\lambda = 2$ **and** $\lambda = -1$

E. $\lambda = 0$ **and** $\lambda = -1$

25. Find the minimum value of $x^2+y^2+z^2$ subject to the constraint $2x+y-z-6=0$.

A. $\frac{25}{6}$

B. 2

C. 4

D. $\boxed{6}$

E. 16

26. A rectangular box is to have volume 48 cubic feet, and is made of three different grades of material. The material for the front and back costs \$1 per square foot, the material for the top and bottom costs \$2 per square foot, and the material for the two ends costs \$3 per square foot. What are the dimensions of the box of minimal cost? Answer: 2 by 4 by 6

27. Find $\left(\frac{\partial w}{\partial y}\right)_x$, that is with x and y independent, at $(w,x,y,z)=(4,2,1,-1)$ if

$$w = x^2y^2 + yz - z^3, \quad x^2+y^2+z^2 = 6$$

A. -1

B. 1

C. 3

D. $\boxed{5}$

E. 7

28. Find cubic approximation of $f(x,y) = \dfrac{1}{1-x-y+xy}$ **near the origin.**

Answer: $1 + x + y + x^2 + xy + y^2 + x^3 + x^2y + xy^2 + y^3$

29. Evaluate $\displaystyle\int_0^{\pi}\int_x^{\pi} \frac{\sin(y)}{y}\,dy\,dx$.

A. -1

B. 0

C. 1

D. $\boxed{2}$

E. $\dfrac{\pi^2}{2}$