THIS EXAM IS CLOSED TO BOOKS AND NOTES. NO CALCULATORS.
Use the back of the **previous** page if more space is needed!

MA303 EXAMINATION III (Practice)

Name______________________________ ID #__________________ Section #______

There are 10 Problems on this booklet. For All problems, mark the answers clearly.

Points awarded

1. (5 pts) ____________
2. (5 pts) ____________
3. (5 pts) ____________
4. (5 pts) ____________
5. (5 pts) ____________
6. (5 pts) ____________
7. (5 pts) ____________
8. (5 pts) ____________
9. (5 pts) ____________
10. (5 pts) ____________

Total Points: ____________

1. (5 points) The approximate value of the solution at $t = 0.2$ of

$$y' = t^3 + y^2, \quad y(0) = 1$$

is evaluated using Euler method with $h = 0.1$. The approximate value is ______________.

2. (5 points) Consider the equation

$$y' = t + y^2, \quad y(1) = 0.$$

The following Runge-Kutta method is used to compute an approximation to $y(3)$ with $h = 2$:

$$y_{n+1} = y_n + \frac{h}{6}(k_{n1} + 2k_{n2} + 2k_{n3} + k_{n4})$$

where

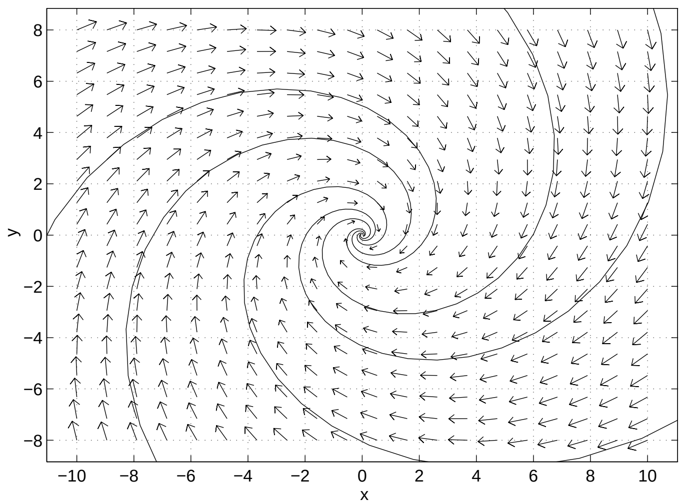
$$k_{n1} = f(t_n, y_n), \quad k_{n2} = f(t_n + \frac{h}{2}, y_n + \frac{h}{2}k_{n1}),$$
$$k_{n3} = f(t_n + \frac{h}{2}, y_n + \frac{h}{2}k_{n2}), \quad k_{n4} = f(t_n + h, y_n + hk_{n3}).$$

The approximate value of $y(3)$ is ______________.

3. (5 points) The phase portrait for a linear system of the form

$$\mathbf{x}' = A\mathbf{x},$$

where A is a 2×2 real matrix, is as follows.



Which of the following best describes the eigenvalues of A:

a) two real negative eigenvalues

b) two real eigenvalues of opposite signs

c) two complex eigenvalues with positive real part

d) two complex eigenvalues with negative real part

e) one negative repeated eigenvalue

4. (5 points) Find all values of k for which the equilibrium point $\begin{pmatrix} 0 \\ 0 \end{pmatrix}$ of the system

$$\begin{pmatrix} x \\ y \end{pmatrix}' = \begin{pmatrix} -2 & k \\ -1 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

is a spiral (in) point.

a) $k < 1$

b) $k > 1$

c) $k < 2$

d) $k > 2$

e) $k = -1$

5. (5 points) If $\mathbf{x}(t) = 4\begin{pmatrix} 1 \\ -1 \end{pmatrix} + 2\left[t\begin{pmatrix} 1 \\ -1 \end{pmatrix} + \begin{pmatrix} 2 \\ -1 \end{pmatrix}\right]$, then $\mathbf{x}(2) =$

a) $\mathbf{x}(2) = \begin{pmatrix} 10 \\ -7 \end{pmatrix}$

b) $\mathbf{x}(2) = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$

c) $\mathbf{x}(2) = \begin{pmatrix} 12 \\ -10 \end{pmatrix}$

d) $\mathbf{x}(2) = \begin{pmatrix} -5 \\ 6 \end{pmatrix}$

e) $\mathbf{x}(2) = \begin{pmatrix} 12 \\ 6 \end{pmatrix}$

6. (5 points) The solution of the initial value problem $\mathbf{x}' = \begin{pmatrix} 2 & 0 \\ -1 & 3 \end{pmatrix}\mathbf{x}$, $\mathbf{x}(0) = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$ is

a) $\mathbf{x}(t) = e^{3t}\begin{pmatrix} 0 \\ 1 \end{pmatrix} + e^{2t}\begin{pmatrix} 1 \\ 1 \end{pmatrix}$

b) $\mathbf{x}(t) = 2e^{3t}\begin{pmatrix} 0 \\ 1 \end{pmatrix} + e^{2t}\begin{pmatrix} 1 \\ 1 \end{pmatrix}$

c) $\mathbf{x}(t) = 3e^{3t}\begin{pmatrix} 0 \\ 1 \end{pmatrix} + e^{2t}\begin{pmatrix} 1 \\ 0 \end{pmatrix}$

d) $\mathbf{x}(t) = 2e^{3t}\begin{pmatrix} 1 \\ 2 \end{pmatrix} - e^{2t}\begin{pmatrix} 1 \\ 0 \end{pmatrix}$

e) $\mathbf{x}(t) = e^{2t}\begin{pmatrix} 1 \\ 3 \end{pmatrix}$

7. (5 points) Given that the matrix $A = \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}$ has complex eigenvalues $r_1 = 1 + i$ and $r_2 = 1 - i$, with corresponding complex eigenvectors $\begin{pmatrix} 1 \\ i \end{pmatrix}$ and $\begin{pmatrix} 1 \\ -i \end{pmatrix}$, find the general solution of $\mathbf{x}' = A\mathbf{x}$.

a) $\mathbf{x} = C_1 e^t \begin{pmatrix} \cos(t) \\ -\sin(t) \end{pmatrix} + C_2 e^t \begin{pmatrix} \sin(t) \\ \cos(t) \end{pmatrix}$

b) $\mathbf{x} = C_1 e^t \begin{pmatrix} \cos(t) \\ -\sin(t) \end{pmatrix} + C_2 e^t \begin{pmatrix} \sin(t) \\ -\cos(t) \end{pmatrix}$

c) $\mathbf{x} = C_1 e^t \begin{pmatrix} \sin(t) \\ \cos(t) \end{pmatrix} + C_2 e^t \begin{pmatrix} \sin(t) \\ -\cos(t) \end{pmatrix}$

d) $\mathbf{x} = C_1 e^t \begin{pmatrix} \cos(t) \\ -\sin(t) \end{pmatrix} + C_2 e^t \begin{pmatrix} \cos(t) \\ \sin(t) \end{pmatrix}$

e) $\mathbf{x} = C_1 e^t \begin{pmatrix} 2\cos(t) \\ \sin(t) \end{pmatrix} + C_2 e^t \begin{pmatrix} \sin(t) \\ 2\cos(t) \end{pmatrix}$

8. (5 points) If the general solution of the homogeneous system $\mathbf{x}' = \begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix} \mathbf{x}$ is given by $\mathbf{x}_H(t) = C_1 e^t \begin{pmatrix} 1 \\ 0 \end{pmatrix} + C_2 e^{2t} \begin{pmatrix} 0 \\ 1 \end{pmatrix}$, then the general solution of the nonhomogeneous system $\mathbf{x}' = \begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix} \mathbf{x} + e^{-t} \begin{pmatrix} -4 \\ -3 \end{pmatrix}$ is

a) $\mathbf{x}(t) = C_1 e^t \begin{pmatrix} 1 \\ 0 \end{pmatrix} + C_2 e^{2t} \begin{pmatrix} 0 \\ 1 \end{pmatrix} + e^{-t} \begin{pmatrix} 4 \\ 3 \end{pmatrix}$

b) $\mathbf{x}(t) = C_1 e^t \begin{pmatrix} 1 \\ 0 \end{pmatrix} + C_2 e^{2t} \begin{pmatrix} 0 \\ 1 \end{pmatrix} + e^{-t} \begin{pmatrix} 4 \\ 0 \end{pmatrix}$

c) $\mathbf{x}(t) = C_1 e^t \begin{pmatrix} 1 \\ 0 \end{pmatrix} + C_2 e^{2t} \begin{pmatrix} 0 \\ 1 \end{pmatrix} + e^{-t} \begin{pmatrix} 1 \\ 0 \end{pmatrix}$

d) $\mathbf{x}(t) = C_1 e^t \begin{pmatrix} 1 \\ 0 \end{pmatrix} + C_2 e^{2t} \begin{pmatrix} 0 \\ 1 \end{pmatrix} + e^{-t} \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

e) $\mathbf{x}(t) = C_1 e^t \begin{pmatrix} 1 \\ 0 \end{pmatrix} + C_2 e^{2t} \begin{pmatrix} 0 \\ 1 \end{pmatrix} + e^{-t} \begin{pmatrix} 2 \\ 1 \end{pmatrix}$

9. (5 points) Consider

$$\mathbf{x}' = \begin{pmatrix} 3 & -2 \\ 2 & -2 \end{pmatrix} \mathbf{x}$$

Find an initial data $\mathbf{x}(0)$ such that when $t \to \infty$, the solution approaches $(0, 0)$.

a) $\mathbf{x}(0) = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

b) $\mathbf{x}(0) = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$

c) $\mathbf{x}(0) = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

d) $\mathbf{x}(0) = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$

e) $\mathbf{x}(0) = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$

10. (5 points) Find the general solution of the system $\mathbf{x}' = \begin{pmatrix} 1 & -4 \\ 4 & -7 \end{pmatrix} \mathbf{x}$

a)

EXAMINATION III (Answer)

1. 1.2211
2. 172
3. D
4. B
5. C
6. B
7. A
8. E
9. D
10. $X(t) = c_1 e^{-3t} \begin{pmatrix} 1 \\ 1 \end{pmatrix} + c_2 e^{-3t} \left(t \begin{pmatrix} 1 \\ 1 \end{pmatrix} + \begin{pmatrix} 1/4 \\ 0 \end{pmatrix} \right)$