

THIS EXAM IS CLOSED TO BOOKS AND NOTES. NO CALCULATORS ARE ALLOWED!
Use the back of the **previous** page if more space is needed!

MA366

EXAMINATION I (Practice)

Fall 2005

Name _____ ID # _____ Section # _____

There are 11 Problems on this booklet. For All problems, show all your work **and** write (or mark) the answers clearly.

Points awarded

1. (5 pts) _____

2. (5 pts) _____

3. (5 pts) _____

4. (5 pts) _____

5. (5 pts) _____

6. (5 pts) _____

7. (5 pts) _____

8. (5 pts) _____

9. (5 pts) _____

10. (5 pts) _____

Total Points: _____

1. (5 points) Choose the properties from the following list: Linear, Separable, 1st order, 2nd order, constant coefficients which describes the equation

a) $(x^2 - y) - (x + 1)y' = 0$: _____, _____.

b) $y' = \frac{2x(y + 1)}{y}$: _____, _____.

c) $y' = xe^{y+x}$: _____, _____.

d) $y'' + 3y' + 4y = 0$: _____, _____.

2. (5 points) The general solution of

$$y' + \frac{2}{x}y = 8x$$

is

a) $y = \frac{4x^2}{1 + 2\ln(x)} + C$

b) $y = 2x^2 + \frac{C}{x^2}$

c) $y = 2x^2 + C$

d) $y = 2x^2 + C\ln(x)$

3. (5 points) The solution of the following initial value problem

$$2xy \frac{dy}{dx} = 1 + y^2, \quad y(1) = -1$$

is

a) $y = -\sqrt{2x - 1}$

b) $y = \sqrt{2x - 1}$

c) $y = e^x$

d) $y = \sqrt{2x}$

4. (5 points) Find the differential equation which has

$$y(x) = c_1 e^{6x} + c_2 x e^{6x}$$

as its general solution.

a) $y'' - 6y' - 36y = 0.$

b) $y'' + 6y' + 36y = 0.$

c) $y'' + xy' + 6y = 0.$

d) $y'' - 12y' + 36y = 0.$

e) $xy'' - 3y' - 12y = 0.$

5. (6 points) If $y'' + 2y' + 5y = 0$, $y(0) = 1$, $y'(0) = 2$, then $y(\frac{\pi}{2}) =$

- a) 1.
- b) 0.
- c) e^π .
- d) $e^{\frac{\pi}{2}}$.
- e) $-e^{-\frac{\pi}{2}}$.

6. (5 points) Which of the following initial data will not guarantee a unique solution for the problem

$$y' = y^{2/3} + (t - 1)^{1/3}, y(t_0) = y_0$$

- a) $(t_0, y_0) = (1, 2)$
- b) $(t_0, y_0) = (2, 0)$
- c) $(t_0, y_0) = (2, 2)$
- d) $(t_0, y_0) = (-1, 2)$

7. (6 points) Find the equilibrium points (solutions) of the differential equation

$$\frac{dy}{dx} = (y - 2)(y - 1)(y - 3)^2, \quad -\infty < y_0 < \infty$$

and classify each one as asymptotically stable, unstable.

8. (5 points) Find the general solution of the following initial value problem

$$(1 - e^x) \frac{dy}{dx} = y + ye^x, \quad y(\ln 2) = -4$$

9. (7 points) At time $t = 0$ a tank contains 2 lb of salt dissolved in 100 gal of water. Assume that fresh water is poured into the tank at a rate of $2 \text{ gal}/\text{min}$, and the mixture is allowed to leave at the same rate. Find the amount of salt in the tank at the end of 20 min .
10. (8 points) The population of mosquitoes in certain area increases at a rate proportional to the current population and, in the absence of other factors, the population doubles each week. There are $500,000$ mosquitoes in the area initially, and predators (birds, etc.) eat $20,000$ mosquitoes per day. Determine the population of mosquitoes in the area at any time.

11.

EXAMINATION I (Practice) (Answer)

1. (a) Linear, 1st order (b) Separable, 1st order (c) 1st order, Separable (d) 2nd order, constant coefficients

2. B

3. A

4. D

5. E

6. B

7. $y=1$, asy. stable; $y=2$, unstable; $y=3$, unstable8. $y(x) = -2e^x / (1 - e^x)^2, x > 0$ 9. $2e^{-2/5}$ lb10. $P(t) = 500e^{t \ln(2)/7} + 140 / \ln(2) (1 - e^{t \ln(2)/7})$ thousand