

THIS EXAM IS CLOSED TO BOOKS AND NOTES. NO CALCULATORS ARE ALLOWED!

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MA366

EXAMINATION II (Practice)

Fall 2005

Name _____ ID # _____ Section # _____

There are 13 Problems on this booklet. For All problems, show all your work **and** write (or mark) the answers clearly.

Points awarded

1. (5 pts) _____

2. (5 pts) _____

3. (5 pts) _____

4. (5 pts) _____

5. (5 pts) _____

6. (5 pts) _____

7. (5 pts) _____

8. (5 pts) _____

9. (5 pts) _____

10. (5 pts) _____

Total Points: _____

1. (3 points) Suppose $u(t) = 3 \sin 3t + 4 \cos 3t$ is a solution to a spring-mass system. The frequency, the period and the amplitude of $u(t)$ are: _____, _____, _____.

2. (5 points) The approximate value of the solution at $t = 3$ of

$$\begin{aligned}y'(t) &= v(t) + t, & y(2) &= 1 \\v'(t) &= -y(t), & v(2) &= -1\end{aligned}$$

is evaluated using Euler method with $h = 0.5$. The approximate value for $y(3)$ is _____.
The approximate value is for $v(3)$ is _____.

3. Find the differential equation which has

$$y(x) = c_1 + c_2 e^{-x} \cos(2x) + c_3 e^{-x} \sin(2x)$$

as its general solution.

a) $y''' - 6y'' - 36y' + 3y = 0.$

b) $y''' - 2y'' - 5y' = 0.$

c) $y''' + 2y'' + 5y' = 0.$

d) $y'' - 2y' + 5y = 0.$

e) $y''' - 3y'' - 12y' = 0.$

4. What is the general solution of $y^{(4)} + 8y'' + 16y = 0$?

a) $y = c_1 e^{2t} + c_2 e^{-2t} + c_3 t e^{2t} + c_4 t e^{-2t}$

b) $y = c_1 e^{2t} + c_2 e^{-2t} + c_3 \cos(2t) + c_4 \sin(2t)$

c) $y = c_1 \cos(2t) + c_2 \sin(2t) + c_3 t \cos(2t) + c_4 t \sin(2t)$

d) $y = c_1 e^t + c_2 e^{-t} + c_3 t e^t + c_4 t^2 e^t$

e) $y = c_1 e^{\sqrt{2}t} + c_2 t e^{\sqrt{2}t}$

5. (5 point)

$$\operatorname{Re}\left(\frac{(2+i)}{(2-i)^2}e^{(-2+3i)t}\right) =$$

a) $\left(\frac{2}{25}\cos(3t) - \frac{9}{25}\sin(3t)\right)e^{-2t}$

b) $\left(\frac{2}{25}\cos(3t) + \frac{1}{25}\sin(3t)\right)e^{-2t}$

c) $\left(\frac{2}{25}\cos(3t) - \frac{11}{25}\sin(3t)\right)e^{-2t}$

d) $\left(-\frac{2}{9}\cos(3t) - \frac{11}{9}\sin(3t)\right)e^{-2t}$

e) $\left(\frac{2}{9}\cos(3t) - \sin(3t)\right)e^{-2t}$

6. (5 points) Determine a suitable form for $Y(t)$ if the method of undetermined coefficients is to be used on

$$y'' + y = t + t \sin(t).$$

a) $Y(t) = At + B + t(Ct + D)\cos(t) + t(Et + F)\sin(t)$

b) $Y(t) = At + B + (Ct + D)(\cos(t) + \sin(t))$

c) $Y(t) = At + B + (Ct + D)\cos(t) + (Et + F)\sin(t)$

d) $Y(t) = At + B + t(Ct + D)(\cos(t) + \sin(t))$

e) $Y(t) = t(At + B) + t(Ct + D)\cos(t) + t(Et + F)\sin(t)$

7. (5 points) A mass weighing 4 lb stretches a spring 2 feet. The mass is pulled 1 foot below equilibrium and released with no initial velocity. Assuming that there is no damping, and that the mass is acted upon by an external force of $2 \cos(wt)$ lb. Find the value of w for which resonance occurs. ($g = 32 \text{ ft/sec}^2$)

- a) 0
- b) 1
- c) 2
- d) 4
- e) no values of w

8. (5 points) If $y'' + 2y' + 5y = 0$, $y(0) = 1$, $y'(0) = 2$, then $y(\frac{\pi}{2}) =$

- a) 0.
- b) $2e^{-\frac{\pi}{2}}$.
- c) e^{π} .
- d) $e^{\frac{\pi}{2}}$.
- e) $-e^{-\frac{\pi}{2}}$.

9. (5 points) Solve the initial value problem :

$$\frac{d^2y}{dt^2} - y = e^{-t}, \quad y(0) = 0, \quad y'(0) = 1.$$

a) $y(t) = \frac{3}{4}e^t - \frac{3}{4}e^{-t} - \frac{1}{2}te^{-t}$

b) $y(t) = \frac{1}{2}e^t - \frac{1}{2}e^{-t} - \frac{1}{2}te^{-t}$

c) $y(t) = e^t - \frac{1}{2}te^{-t}$

d) $y(t) = \sin(t) - \frac{1}{2}t^2e^{-t}$

e) $y(t) = -te^{-t}$

10. (5 points) Find **a** solution (simplify as much as possible) of

$$z'' + 2z' - 3z = 5e^{(-2+3i)t}$$

11. (6 points) Using variation of parameters to find the general solution of

$$\frac{d^2y}{dt^2} + 4y = \frac{2}{\cos 2t}$$

12. (6 points) Given $y_1(t) = t^3$ is a solution of

$$t^2y'' - 6y = 0.$$

Find the other solution $y_2(t)$ which is linearly independent of y_1 . (Hint: $y_2(t)$ can be found by assuming $y(t) = t^3 * v(t)$ is a solution and solve for $v(t)$. This technique is called *reduction of order*.) You can follow the following steps:

- a) Substitute $y(t) = t^3 * v(t)$ into the equation and find the equation for $v'(t)$ (involving $v''(t)$ and $v'(t)$).
- b) Let $u(t) = v'(t)$ and solve for $u(t)$ which is a separable equation.
- c) Integrate to get $v(t)$ and then obtain $y_2(t)$.

13.

EXAMINATION II (Practice) (Answer)

1. $3, \frac{2}{3}\pi, 5$

2. $2, -2.25$

3. C

4. C

5. C

6. A

7. D

8. E

9. A

10. $z = \left(-\frac{1}{3} + \frac{1}{6}i\right)e^{(-2+3i)t}$

11. $y(t) = \left(\frac{1}{2} \ln |\cos(2t)| + c_1\right) \cos(2t) + (t + c_2) \sin(2t)$

12. (a) $6v' + tv'' = 0$ (b) $u(t) = v'(t) = C_1t^{-6}$ (c) $v(t) = D_1t^{-5}$ $y_2(t) = t^{-2}$