

INTRODUCTION TO *dfield* , *pplane* AND *fplot*

It might be reasonable to expect that if two solutions of a differential equation which start close together, then they should stay relatively close together. We can investigate the behavior of solutions using the routine *dfield* .

- (1) Use *dfield6* and the “Keyboard Input” feature to plot the solution to the differential equation $\mathbf{y}' = \mathbf{y}^2 - 2\mathbf{t}$ with $\mathbf{y}(0) = \mathbf{0.9185}$. Now using the “Zoom In” feature, approximate $y(3.2)$ to within 2 decimal places.
- (2) Erase all solutions. Now use *dfield6* and the “Keyboard Input” feature to plot the solution to the differential equation $\mathbf{y}' = \mathbf{y}^2 - 2\mathbf{t}$ with $\mathbf{y}(0) = \mathbf{0.9184}$ (instead of $y(0) = 0.9185$). Use the “Zoom In” feature to approximate $y(3.2)$ to within 2 decimal places. Is this approximation close to the approximation in (1) ?
- (3) Erase all solutions and using *dfield6* plot the two solutions to:

$$\left\{ \begin{array}{l} y' = y^2 - 2t \\ y(0) = 0.9185 \end{array} \right. \quad \text{and} \quad \left\{ \begin{array}{l} y' = y^2 - 2t \\ y(0) = 0.9184 \end{array} \right.$$

on a single graph. This shows the solutions start close. Do they remain close as $t \rightarrow \infty$?

Remark: This example illustrates *Chaos Theory*: a small change in the initial condition can lead to very different outcomes. This specific example shows that if you are a mere 0.0001 off in your measurement of $y(0)$, you could arrive at drastically different solutions. This is one reason why weather prediction for more than a few days into the future is so difficult.

- (4) The routine *pplane6* is similar to *dfield6*, except it is used to sketch solutions to *systems* of differential equations of the form

$$\frac{dx}{dt} = F(t, x, y)$$

$$\frac{dy}{dt} = G(t, x, y)$$

Use *pplane* and the Keyboard Input feature to plot solutions to the system

$$x' = 2y + 1$$

$$y' = -\frac{3}{1+x}$$

with $x = 0$ and various values of $y = y_0$. Estimate the *smallest* value of y_0 so that when $x = 0$ and $y = y_0$, the solution satisfies $y \rightarrow \infty$ (i.e., the solution y is unbounded).

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(5) The solution to the initial value problem

$$y' = \frac{(1 - 2x)}{e^{2x}} + 3x^2 - \pi \sin \pi x, \quad y(0) = 2$$

is given by

$$y = xe^{-2x} + x^3 + \cos \pi x + 1.$$

Use **fplot** to determine the value(s) of x (correct within 2 decimal places) so that

$$y = \mathbf{seed}.$$

Use **dfield** to check you answer.

(6) **Challenge problem:** The solution $y = y(x)$ to the initial value problem

$$\frac{dy}{dx} = \frac{2y - 3x^2}{2(y - x)}, \quad y(1) = 0$$

is given (implicitly) by

$$x^3 + y^2 - 2xy = 1.$$

Approximate $y(-1)$ correct to 2 decimal places using:

- (a) **dfield**
- (b) **fplot**

Even though **pplane** is used for systems of differential equations (and for higher order differential equations as you'll see later), one can use **pplane** to find solutions to a single differential equation.

- Use **dfield**, with window $-1 \leq t \leq 1$, $-1 \leq x \leq 1$, to plot the solutions of the differential equation $x' = 2xe^t - t^2$ with these initial conditions:

$$(t, x) = (0, -1), (0, -0.6), (0, 0), \text{ and } (0, \frac{1}{2}).$$

Print one graph with all 4 solutions on it.

- Now use **pplane** to plot solutions to the system

$$\begin{aligned} y' &= 1 \\ x' &= 2xe^y - y^2 \end{aligned}$$

(make sure $y' = 1$ is the first equation, else the axes will be reversed on your graph) using the window $-1 \leq y \leq 1$, $-1 \leq x \leq 1$ and initial conditions:

$$(y, x) = (0, -1), (0, -0.6), (0, 0), \text{ and } (0, \frac{1}{2}).$$

Do these solutions match those above? Can you explain why?