MA 16200 Exam 1 Study Guide, Fall 2014

LESSON 1

Chapter 12.1 3-dimensional coordinate system

1. Distance between points (x_1, y_1, z_1) and (x_2, y_2, z_2) is $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$.
2. Sphere with center (h, k, l) and radius r is $(x - h)^2 + (y - k)^2 + (z - l)^2 = r^2$.

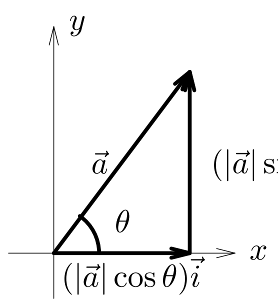
Chapter 12.2 Vectors

1. Vector from points $A(a_1, a_2, a_3)$ to $B(b_1, b_2, b_3)$ is $\overrightarrow{AB} =< b_1 - a_1, b_2 - a_2, b_3 - a_3 >$.
2. Sketch linear combinations of vectors. For example, given vectors $\vec{a}$ and $\vec{b}$ and scalars c and d, sketch $c\vec{a}$, sketch $\vec{b} - \vec{a}$, sketch $c\vec{b} + d\vec{a}$.
3. Magnitude of $\vec{a} =< a_1, a_2, a_3 >$ is $|\vec{a}| = \sqrt{a_1^2 + a_2^2 + a_3^2}$.

LESSON 2

Chapter 12.2 Vectors

1. Unit vectors $\vec{i} =< 1, 0, 0 >$, $\vec{j} =< 0, 1, 0 >$, and $\vec{k} =< 0, 0, 1 >$. $\quad < a, b, c >= a\vec{i} + b\vec{j} + c\vec{k}$.
2. Let $\vec{a} = a_1\vec{i} + a_2\vec{j}$ be a two dimensional vector represented with initial point at the origin. If θ is the angle from the positive x–axis to $\vec{a}$, then $\vec{a} = a_1\vec{i} + a_2\vec{j} = (|\vec{a}| \cos\theta)\vec{i} + (|\vec{a}| \sin\theta)\vec{j}$.



LESSON 3

Chapter 12.3 Dot product

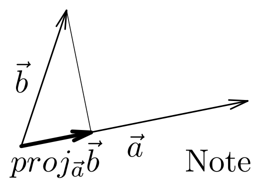
1. The dot product of $\vec{a} =< a_1, a_2, a_3 >$ and $\vec{b} =< b_1, b_2, b_3 >$ is $\vec{a} \cdot \vec{b} = a_1b_1 + a_2b_2 + a_1b_3$.
2. If θ is the angle between $\vec{a}$ and $\vec{b}$ then $\cos\theta = \dfrac{\vec{a} \cdot \vec{b}}{|\vec{a}||\vec{b}|}$.

   $\vec{a}$ and $\vec{b}$ are perpendicular if and only if $\vec{a} \cdot \vec{b} = 0$.
3. Properties of the dot product. (see page 779 in the text)
4. Direction cosines. If $\vec{a} = \langle a_1, a_2, a_3 \rangle$, then $\langle \cos\alpha, \cos\beta, \cos\gamma \rangle = \left\langle \dfrac{a_1}{|\vec{a}|}, \dfrac{a_2}{|\vec{a}|}, \dfrac{a_3}{|\vec{a}|} \right\rangle$.

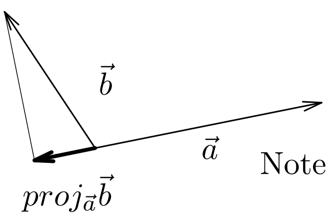
5. The vector projection of $\vec{b}$ onto $\vec{a}$ is a multiple of $\dfrac{\vec{a}}{|\vec{a}|}$, the unit vector in the direction of $\vec{a}$.

   Vector projection of $\vec{b}$ onto $\vec{a}$ is $proj_{\vec{a}}\vec{b} = \left(\dfrac{\vec{a}\cdot\vec{b}}{|\vec{a}|}\right)\dfrac{\vec{a}}{|\vec{a}|}$.

   Scalar projection of $\vec{b}$ onto $\vec{a}$ is $comp_{\vec{a}}\vec{b} = \dfrac{\vec{a}\cdot\vec{b}}{|\vec{a}|}$. $comp_{\vec{a}}\vec{b}$ is the "signed" length of $proj_{\vec{a}}\vec{b}$.

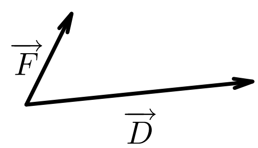


Note that $comp_{\vec{a}}\vec{b} > 0$



Note that $comp_{\vec{a}}\vec{b} < 0$

6. The work done by a constant force $\overrightarrow{F}$ is the dot product $\overrightarrow{F}\cdot\overrightarrow{D}$ where $\overrightarrow{D}$ is the displacement vector.



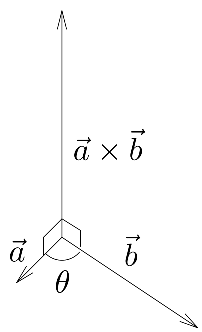
LESSON 4

Chapter 12.4 Cross product

1. If $\vec{a} = a_1\vec{i} + a_2\vec{j} + a_3\vec{k}$ and $\vec{b} = b_1\vec{i} + b_2\vec{j} + b_3\vec{k}$

   then $\vec{a} \times \vec{b} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} =< a_2b_3 - a_3b_2, a_3b_1 - a_1b_3, a_1b_2 - a_2b_1 >.$

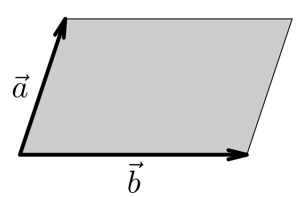
   $\vec{a}$, $\vec{b}$ and $\vec{a} \times \vec{b}$ form a right–hand system.
   $\vec{a} \times \vec{b}$ is perpendicular to both $\vec{a}$ and $\vec{b}$.



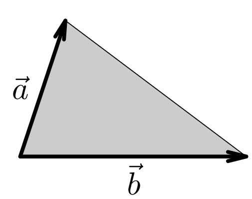
2. If θ is the angle between $\vec{a}$ and $\vec{b}$, then $|\vec{a} \times \vec{b}| = |\vec{a}||\vec{b}| \sin\theta$.

3. Properties of the cross product. (see page 790 of the text)

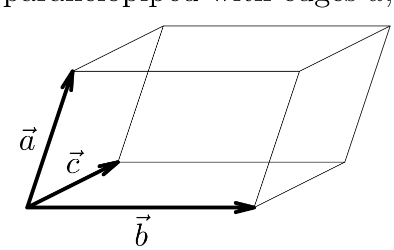
4. $|\vec{a} \times \vec{b}|$ is the area of the parallelogram determined by $\vec{a}$ and $\vec{b}$.



   $\dfrac{|\vec{a} \times \vec{b}|}{2}$ is the area of the triangle determined by $\vec{a}$ and $\vec{b}$.



5. $\left|\vec{a} \cdot (\vec{b} \times \vec{c})\right|$ is the volume of the parallelopiped with edges $\vec{a}$, $\vec{b}$ and $\vec{c}$.



LESSON 5

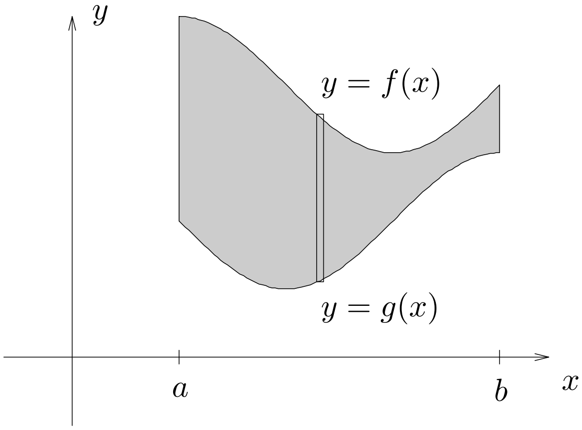
Chapter 6.1 Area

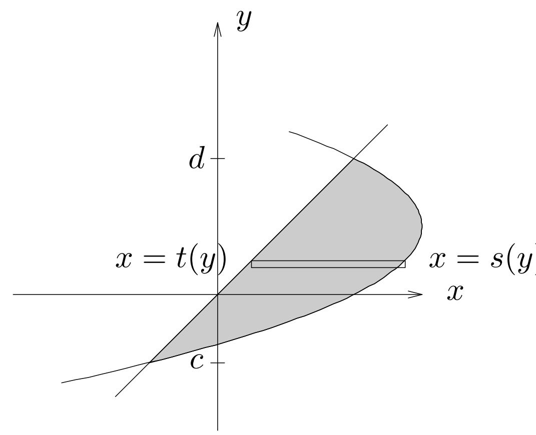
Example on **Left**: Let a plane region R be bounded by $y = f(x)$, $y = g(x)$, $x = a$ and $x = b$. (see figure below).

Example on **Right**: Let a plane region R be bounded by $x = s(y)$, $x = t(y)$, $y = c$ and $y = d$. (see figure below).

$$\text{Area} = \int_a^b \text{(length of slice)(thickness of slice)}$$

$$\int_a^b (f(x) - g(x))\ dx \qquad\qquad \int_c^d (s(y) - t(y))\ dy$$





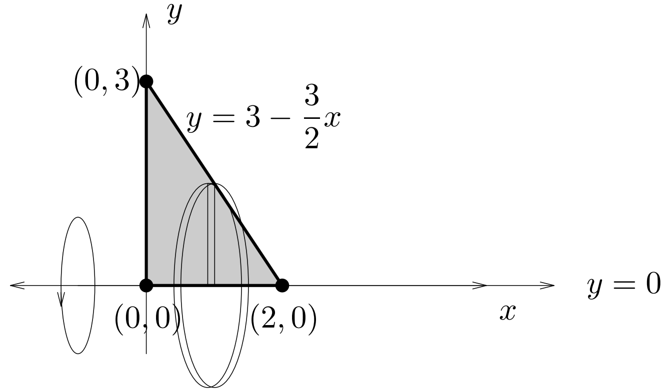
LESSON 6

Chapter 6.2 Volumes by cross section

Volumes by disks:

Always slice plane region PERPENDICULAR to the rotating axis

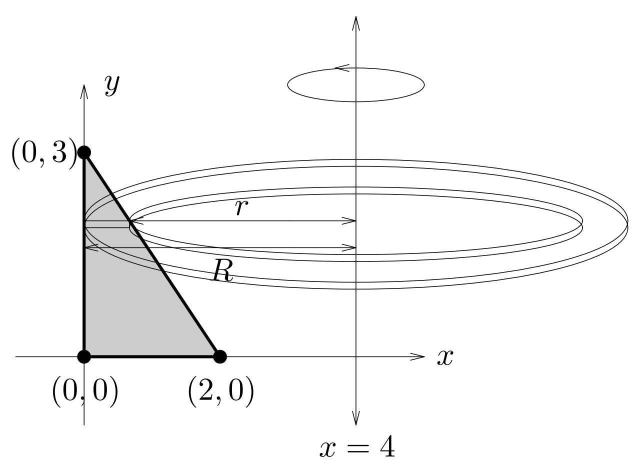
$$\text{Volume} = \int_a^b \pi(\text{radius})^2 \ (\text{thickness of slice}) = \int_0^2 \pi \left(\left(2 - \frac{2}{3}y\right) - (0)\right)^2 \ dx$$



Volumes by washers:

Always slice plane region PERPENDICULAR to the rotating axis

$$\text{Volume} = \int_a^b \pi(R^2 - r^2) \ (\text{thickness of slice}) = \int_0^3 \pi \left((4-0)^2 - (4 - \left(2 - \frac{2}{3}y\right)^2\right) \ dy$$

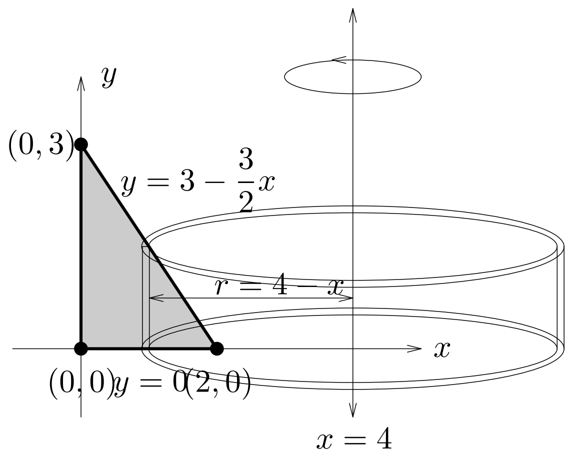


LESSON 7

Chapter 6.3 Volumes by cylindrical shells

Always slice plane region PARALLEL to the rotating axis

$$\text{Volume} = \int_a^b 2\pi(\text{radius})(\text{Length of slice})\ (\text{thickness of slice}) = \int_0^2 2\pi(4-x)\left(\left(3-\frac{3}{2}x\right)-(0)\right)\ dx$$



LESSON 8

Chapter 6.4 Work

If a constant force F is exerted in moving a object a distance D along a line, then the work W done is $W = FD$. The units in the Metric and English systems are given below.

Quantity	*EnglishSystem*	*MetricSystem*
Mass m	slug (=lb–sec^2/ft)	kilogram kg
Force F	pounds (lbs)	Newtons N (=kg–m/sec^2)
Distance d	feet	meters m
Work W	ft–lbs	Joules J (=kg–m^2/sec^2)
g	32 ft/sec^2	9.8 m/sec^2

Hooke's Law: The force f required to maintain a spring stretched x units beyond its natural length is proportional to x: $f(x) = kx$.

The Work W required to stretch a spring n units beyond its natural length is $W = \displaystyle\int_0^n kx\ dx$.

Lifting water (or a liquid) to the top of a tank.

Chapter 6.5 Average value of a function

The average value of a function f on the interval $[a, b]$, f_{ave}, is given by $f_{ave} = \dfrac{1}{b-a}\displaystyle\int_a^b f(x)\ dx$.

LESSON 9

Chapter 7.1 Integration by parts

Let $u = f(x)$ and $v = g(x)$.

$$\int f(x)g'(x)\ dx = f(x)g(x) - \int g(x)f'(x)\ dx$$

which is the same as

$$\int u\ dv = uv - \int v\ du$$

LIATE: Choose u to be the left-most function in the list: L$^{\text{og}}$ I$^{\text{nversetrig}}$ A$^{\text{lgebraic}}$ T$^{\text{rig}}$ E$^{\text{xponential}}$

$$\int_a^b u\ dv = uv\Big|_a^b - \int_a^b v\ du$$
