MA 16200 Study Guide for material since Exam 3, Fall 2014

NOTE: SINCE THE FINAL EXAM WILL COVER ALL THE MATERIAL OF THE COURSE, YOU SHOULD ALSO CONSULT THE EXAM SUMMARIES FOR EXAMS 1, 2 AND 3

LESSON 30 Chapter 11.10 Binomial Series

$$(1+x)^k = \sum_{n=0}^{\infty} \binom{k}{n} x^n = \sum_{n=0}^{\infty} \frac{k!}{n!(k-n)!)} x^n = 1 + kx + \frac{k(k-1)}{2!}x^2 + \frac{k(k-1)(k-2)}{3!}x^3 + \cdots$$

where k any real number and $|x| < 1$

LESSON 31 Chapter 10.1 Curves defined by parametric equations

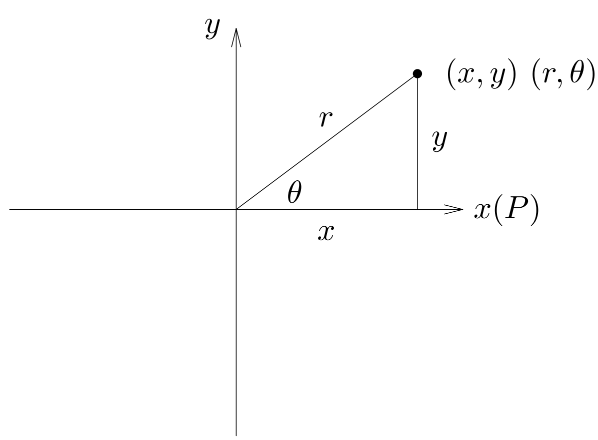
- $x = f(t), \quad y = g(t)$ are called parametric equations and t is called the parameter.
- As t varies $(x, y) = (f(t), g(t))$ varies and traces out a curve C called a parametric curve.
- If you think of t as time, then parametric equations not only describe a set of points (the graph) but also the "time" a particle is at each point on the graph. Parametric equations not only describe the "race track" but also the "race car" on the track!
- The Pythagorean identity $sin^2 t + \cos^2 t = 1$ is a useful identity to elminate the parameter for circular curves. Ex: $x = 3\sin t, \quad y = 3\cos t \;\rightarrow\; x^2 + y^2 = 9$.

LESSON 32 Chapter 10.1 Calculus with parametric curves

- Given parametric equations $x = f(t), \quad y = g(t)$, the derivative $\dfrac{dy}{dx} = \dfrac{\frac{dy}{dt}}{\frac{dx}{dt}}$ and the second derivative,

$$\frac{d^2y}{dx^2} = \frac{\frac{dy/dx}{dt}}{\frac{dx}{dt}}.$$

LESSONS 33 and 34 Chapter 10.3 Polar Coordinates



- The Polar coordinate system consists of a point called the Pole (corresponding to the origin) and the Polar Axis (corresponding to the positive x-axis). A polar point (r, θ) is plotted by thinking of rotating the polar axis θ radians (counterclockwise if $\theta > 0$ and clockwise is $\theta < 0$) and then plotting the point r units along the rotated Polar axis if $r > 0$, and along the Polar axis extended through the Pole if $r < 0$. Each point in the polar plane has an infinite number of different (r, θ) coordinates (while each point in the rectangular plane has a unique (x, y) coordinate).
- Polar to Rectangular convertion equations. $x = r\cos\theta, \quad y = r\sin\theta.$
- Rectangular to Polar convertion equations. $r^2 = x^2 + y^2, \quad \theta = \arctan\left(\frac{y}{x}\right).$
- Basic Polar Curves:

  Circles $r = a\sin\theta, \quad r = a\cos\theta$

  Cardioids $r = a + b\sin\theta, \quad r = a + b\cos\theta$

  Rose Curves $r = a\sin(n\theta), \quad r = a\cos(n\theta).$

  $$n \text{ even} \to 2n \text{ leaves}, \quad n \text{ odd} \to n \text{ leaves}.$$

  Spirals $r = \theta$ (spirals out from pole with increasing θ) and $r = \dfrac{1}{\theta}$ (spirals in toward pole with increasing θ).

LESSON 35 Conic Sections

- **Parabola**

  $4py = x^2$ has vertex $(0, 0)$, focus $(0, p)$ and directrix $y = -p$.

  $4px = y^2$ has vertex $(0, 0)$, focus $(p, 0)$ and directrix $x = -p$.

- **Ellipse**

  $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$ has center $(0, 0)$.
  Ellipse has major axis (longer axis) along x-axis if $a^2 > b^2$.
  Ellipse has major axis (longer axis) along y-axis if $a^2 < b^2$.
  Vertices are endpoints of major axis.
  Foci are on major axis, c units from center, where $c^2 = a^2 - b^2$ (if $a^2 > b^2$), or $c^2 = b^2 - a^2$ (if $b^2 > a^2$).

- **Hyperbola**

  - $\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1$
    has center $(0, 0)$, vertices $(a, 0)$ and $(-a, 0)$,
    foci $(c, 0)$ and $(-c, 0)$, where $c^2 = a^2 + b^2$, and branches open left and right.
    Asymptotes are $y = \pm\dfrac{b}{a}x$
  - $\dfrac{y^2}{b^2} - \dfrac{x^2}{a^2} = 1$
    has center $(0, 0)$, vertices $(0, b)$ and $(0, -b)$,
    foci $(0, c)$ and $(0, -c)$, where $c^2 = a^2 + b^2$, and branches open up and down.
    Asymptotes are $y = \pm\dfrac{b}{a}x$

LESSON 36 Appendix H Complex Numbers

Let $\sqrt{-1} = i$. Note that $i^2 = (-i)^2 = -1$, so we call i the principal square root of i.

addition: $(a + bi) + (c + di) = (a + c) + (b + d)i$

subtraction: $(a + bi) - (c + di) = (a - c) + (b - d)i$

multiplication: $(a + bi)(c + di) = (ac - bd) + (ad + bc)i$

Conjugate of $a + bi$ is $\overline{a + bi} = a - bi$

division: $\dfrac{a + bi}{c + di} = \dfrac{(a + bi)}{(c + di)}\dfrac{(c - di)}{(c - di)}$

Modulus of $a + bi$ is $|a + bi| = \sqrt{a^2 + b^2}$

Polar form of a complex number $z = a + bi$. Let θ be the angle between the positive Real axis and the line joining z and the origin. Then $z = |z|(\cos\theta + i\sin\theta)$.