

Assume that $f \in L^1(0, 2\pi)$ is a periodic function of period 2π , and let $\{s_n\}$ denote the sequence of the partial sums of the Fourier series generated by f . Show that

$$s_n(x) = \frac{2}{\pi} \int_0^\pi \frac{f(x+t) + f(x-t)}{2} D_n(t) dt,$$

where $D_n(t) = \frac{\sin((n+1/2)t)}{2 \sin t/2}$ if $t \neq 2\pi m$, m integer; and $D_n(t) = n + 1/2$ if $t = 2\pi m$, m integer.