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Isoperimetric and trace inequalities with respect to Carnot-Carathéodory metrics.

Geometry Seminars, 1996–1997 (*Italian*) (*Bologna*), 51–62, *Univ. Stud. Bologna*, *Bologna*, 1998.

This note focuses on the problem of existence of traces for Sobolev spaces associated to a family of locally Lipschitz real vector fields $\{X_1, \cdots, X_m\}$ in $\mathbf{R}^n$. Denote by d the Carnot-Carathéodory metric associated to the system $X_1, \cdots, X_m$. The vector fields must satisfy the following three conditions: (H1) The identity map from $\mathbf{R}^n$ equipped with the Euclidean metric into $\mathbf{R}^n$ equipped with the Carnot-Carathéodory metric is continuous. (H2) The balls of the Carnot-Carathéodory metric satisfy a doubling inequality with respect to the Lebesgue metric. (H3) There exists a weak-type Poincaré inequality with respect to the gradient associated to the $X_1, \cdots, X_m$.

Examples are given by vector fields satisfying the Hörmander finite rank condition, by the so-called Grushin-Baouendi vector fields and by the (Lipschitz) vector fields associated to the subelliptic operators studied by Fefferman and Phong.

The main theorem states: Let f be a function whose weak gradient in the X_i directions has bounded L^p norm in an open set Ω. Then f has an L^p restriction to the boundary of Ω when the surface balls have the correct rate of growth. In the particular case of the Heisenberg group this happens for every C^2 set.

Several extensions and applications are mentioned in the paper, together with an outline of the proofs.

{For the entire collection see 99b:00019} *Luca Capogna* (1-AR)