Results from MathSciNet: *Mathematical Reviews* on the Web
© Copyright American Mathematical Society 2004

96d:46032 46E35 26D20 35J99 58G03

Capogna, Luca (1-PURD); **Danielli, Donatella** (1-PURD);
Garofalo, Nicola (1-PURD)

The geometric Sobolev embedding for vector fields and the isoperimetric inequality.

Comm. Anal. Geom. **2** (1994), *no.* 2, 203–215.

The authors prove a generalization of the usual Sobolev inequalities. Instead of the standard gradient, they consider a family χ of vector fields. There corresponds to χ a metric $d_\chi(x,y)$ and this in turn defines the "generalized" balls $B(x,r)$. With everything defined as one would expect, the authors then make two assumptions: Let $U\subset\mathbf{R}^n$ be a bounded set and suppose there exist constants C_1, C_2, and R_0 such that, whenever $R\le R_0$ and $x\in U$, then (A) $|B(x,2R)|\le C_1|B(x,R)|$, and

$$\text{(B)}\qquad |u(y)|\le C_2\int_{B(x,R)}|D_\chi u(\xi)|\frac{d_\chi(y,\xi)}{|B(y,d_\chi(y,\xi))|}\,d\xi.$$

This paper gives concrete situations where these assumptions can be verified.

Theorem 1.1: Assume (A) and (B); then

$$\left(\frac{1}{|B_R|}\int_{B_R}|u|^k\,dx\right)^{1/k}\le C_3R\left(\frac{1}{|B_R|}\int_{B_R}|D_\chi u|\,dx\right),$$

where $1\le k\le Q/(Q-1)$, while $Q=(\log C_1)/(\log 2)$.

A standard transformation now produces the fully decorated Sobolev inequalities. The method of proof involves first establishing a suitable isoperimetric inequality. For the classical Sobolev inequalities the use of the isoperimetric inequality goes back to G. Talenti [Ann. Mat. Pura Appl. (4) **110** (1976), 353–372; MR **57** #3846], and in fact well before that—at least to the early sixties (in unpublished notes by Eugene Rodemich). *James E. Ross* (1-SDS)