2002f:46049 46E35 35H20

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Sub-elliptic Besov spaces and the characterization of traces on lower dimensional manifolds.

Harmonic analysis and boundary value problems (*Fayetteville, AR*, 2000), 19–37, *Contemp. Math.*, 277, *Amer. Math. Soc.*, *Providence*, *RI*, 2001.

In this paper Besov spaces with respect to a non-negative Borel measure μ are introduced. These spaces play a fundamental role in the study of boundary value problems in classical analysis. Let μ be a non-negative Borel measure supported on a closed set $F \subseteq \mathbb{R}^n$. Assume $1 \le p < \infty,\ 0 < \beta < 1$, and $s > 0$; then the Besov space $B^p_\beta(F, d\mu)$ is introduced as the set of all $f \in L_p(F, d\mu)$ such that the semi-norm

$$\mathcal{N}^p_\beta(f, F, d\mu) = \left(\int_F \int_F \left(\frac{|f(x)-f(y)|}{d(x,y)^\beta}\right)^p \frac{d(x,y)^s}{|B(x,d(x,y))|}\ d\mu(y)d\mu(x)\right)^{\frac{1}{p}}$$

is finite. Here $B(x,r)$ are the usual (open) balls in $\mathbb{R}^n$ with respect to the Carnot-Carathéodory metric $d(x,y)$ corresponding to a system $X = \{X_1, \dots, X_m\}$ of C^∞ vector fields satisfying the finite rank condition. The Besov space $B^p_\beta(F, d\mu)$ is normed in the usual way, i.e.

$$\|f\|_{B^p_\beta(F,d\mu)} = \|f\|_{L_p(F,d\mu)} + \mathcal{N}^p_\beta(f, F, d\mu).$$

The authors then state trace and extension theorems of the following type. Assume that, in addition, the measure μ with $\operatorname{supp}\mu = F,\ \mu(F) > 0$, satisfies some (upper) growth conditions of Ahlfors type (relating s and μ); then for sufficiently small β and balls B_0 there is a number $\sigma > 0$ such that $\|f\|_{B^p_\beta(F,d\mu)} \le C\,\|f\|_{\mathcal{L}^{1,p}(\sigma B_0, dx)}$, where $\mathcal{L}^{1,p}(\Omega, dx)$ denotes the first-order subelliptic Sobolev spaces related to the system X. Furthermore, when Ω is an (ε,δ)-domain and $\operatorname{supp}\mu \subseteq \partial\Omega$, conditions are given such that there is a linear and bounded trace operator $\mathrm{Tr}\colon \mathcal{L}^{1,p}(\Omega, dx) \to B^p_\beta(\partial\Omega, d\mu)$. Conversely, for $1 \le p < \infty$, sufficiently small $s > 0$ and $F = \operatorname{supp}\mu$ compact with $|F| = 0$, there is a bounded linear extension operator $\mathcal{E}$ from $B^p_{1-\frac{s}{p}}(F, d\mu)$ into $\mathcal{L}^{1,p}(\Omega, dx)$. Further interesting consequences for trace and extension operators are derived. There is also an embedding theorem of Sobolev type, $B^p_\beta(\Omega, d\mu) \subset L^q(\Omega, d\mu)$. This is followed by some important examples of such settings as described above (concerning the particular underlying domain and μ) where the geometric setting is that of

Carnot groups. Finally, there is a short discussion on the subelliptic Neumann problem.

{For the entire collection see 2001m:00022}

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