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Formules de représentation et théorèmes d'inclusion pour des opérateurs sous-elliptiques. (French. English summary) [Representation formulas and embedding theorems for subelliptic operators]

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This note presents some Sobolev-type inequalities for vector fields satisfying Hormander's condition for hypoellipticity. The author considers some smooth vector fields $X_1, \cdots, X_m$ in $\mathbf{R}^n$ satisfying rank $\mathrm{Lie}[X_1, \cdots, X_m] = n$; these vector fields define a possibly degenerate metric d whose ball with radius R and center x_0 will be denoted by $B = B(x_0, R)$. The operator $\mathfrak{L} = \sum X_j^* X_j$ is such that $\mathfrak{L}u = \mathrm{div}(A\nabla u)$ and the subelliptic gradient $D_{\mathfrak{L}}$ is such that $|D_{\mathfrak{L}}u|^2 = A\nabla u \cdot \nabla u$. The author claims that the following inequalities are true:

$$\left[\frac{1}{\omega(B)}\int_B |u(x)|^{kp}\omega(x)dx\right]^{1/kp} \leq C|B|^{1/Q}\left[\frac{1}{\omega(B)}\int_B |D_{\mathfrak{L}}u(x)|^p\omega(x)dx\right]^{1/p},$$

where $p > 1$, ω is an A_p weight and Q is the homogeneous dimension for $\mathfrak{L}$ at x_0, $1 \leq k \leq \delta + Q/(Q-1)$, $u \in C_0^\infty(B)$, and

$$\left[\int_B |u(x)|^q dx\right]^{1/q} \leq C\left[\int_B |D_{\mathfrak{L}}u(x)|^p dx\right]^{1/p}$$

with $1 < p < Q$, $1/q = 1/p - 1/Q$.

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