# Asymptotic analysis of an optimal location problem.

One considers the problem of optimal location of masses(say production centers) in order to approximate a given density $f$ on a domain $\Omega$ in $\mathbb{R}^d$ (say demand for a certain commodity). A criterion for optimization is the average distance to the centers of production

$$\int_\Omega d(x,\Sigma)f(x)\ dx$$

where $\Sigma = \{x_1, x_2, ..., x_n\}$ is a given location of production centers and $d(x,\Sigma)$ is the distance of any point $x$ to the nearest production center. We prove, for $n$ large, that [1, 2]

$$\inf_\Sigma \left\{ \int_\Omega d(x,\Sigma)f(x)\ dx : \sharp\Sigma \leq n \right\} \sim n^{-1/d} c_d \left( \int_\Omega f^{d/(d+1)} \right)^{(d+1)/d}$$

where $c_d$ is a constant which does not depend on $\Omega$ or $f$. In fact it is given by

$$c_d = \lim_{n\to\infty} n^{1/d} \inf \left\{ \int_Q d(x,\Sigma)\ dx : \Sigma \subset Q\, ,\ \sharp\Sigma \leq n \right\}$$

where $Q$ is the unit cube $[0,1]^d$ in $\mathbb{R}^d$. Some generalizations and open questions will be discussed.

## References


[1] G. Bouchitté, C. Jimenez, M. Rajesh: Asymptotique d'un problème de positionnement optimal *C.R. Acad. Sci. Paris* Ser. I **335** (2002), 1-6.

[2] J. A. Bucklew, G. Wise: Multidimensional asymptotic quantization theory with $r$th power measures. IEEE Trans. Inf. Theory **28**(1982), 239-247.