

MA 262 - Midterm 1
February 7, 2002
Prof. D. Danielli

Name.....

I. D. no. Division

Problem	Score	Max. pts.
1		10
2		10
3		10
4		10
5		10
6		10
7		10
8		10
9		10
10		10
Total		100

No calculators, books, or notes may be used.
This exam contains 10 pages. Please make sure you have all of them.

Time: 75 minutes.

1. Find the general solution of the differential equation

$$y' + \left(\frac{1}{x} - 1\right)y = \frac{2}{x}.$$

2. Solve the initial value problem

$$\frac{dy}{dx} = \frac{3x^2 + 4x}{2(y - 1)}, \quad y(0) = -1.$$

3. If $y_1(t) = e^t$ is a solution of the equation

$$ty'' - (1+t)y' + y = 0,$$

find a second linearly independent solution.

4. In a culture a certain strain of bacteria is growing at a rate which is proportional to the number present. Initially there were 10 bacteria in the culture. Exactly 24 hours later there are 25 bacteria. What will be the number of bacteria after 10 days?

5. Find the general solution of the differential equation

$$y'' - 8y' + 16y = 20 \cos 2t.$$

6. Find the general solution of the differential equation

$$t^2 y' + 2ty = y^3, \quad t > 0.$$

7. Find the general solution of the differential equation

$$\left((\sec x)^2 + 3x^2 \tan y - \frac{2y^3}{x^3} \right) dx + \left(x^3 (\sec y)^2 + 4y^3 + \frac{3y^2}{x^2} \right) dy = 0.$$

8. Determine the motion of the spring-mass system governed by the initial value problem

$$\frac{d^2y}{dt^2} + 2\frac{dy}{dt} + y = 0, \quad y(0) = -1, \quad \frac{dy}{dt}(0) = 2.$$

State whether the motion is underdamped, critically damped, or overdamped. Find the time at which the mass passes through the equilibrium position, and make a sketch depicting the motion.

9. Find the general solution of the differential equation

$$\frac{dy}{dx} = \frac{x^2 + 3y^2}{2xy}.$$

10. A murder victim is discovered at midnight and the temperature of the body is recorded at 31° C. One hour later, the temperature of the body is 29° C. Assume that the surrounding air temperature remains constant at 21° C. Use Newton's law of cooling to calculate the victim's time of death.
(*Note:* The "normal" temperature of a living human is approximately 37° C.)