MA 262 - Midterm 1
March 21, 2002
Prof. D. Danielli

Name..

I. D. no. .... Division ....

Problem	Score	Max. pts.
1		10
2		10
3		10
4		10
5		10
6		10
7		10
8		10
9		10
10		10
Total		100

No calculators, books, or notes may be used.
This exam contains 10 pages. Please make sure you have all of them.

Time: 75 minutes.

1. Consider the system $\mathbf{Ab} = \mathbf{x}$, where

$$\mathbf{A} = \begin{pmatrix} -2 & 0 & 3 \\ -1 & 1 & 0 \\ 3 & 1 & -2 \end{pmatrix},$$

and $\mathbf{b} = \begin{pmatrix} -1 \\ 3 \\ 2 \end{pmatrix}$. Use row-echelon reduction and back substitution to solve the system.

2. Use reduced row-echelon reduction to find $\mathbf{A}^{-1}$, where

$$\mathbf{A} = \begin{pmatrix} 1 & -1 & -2 \\ 3 & 0 & -3 \\ 2 & 0 & 1 \end{pmatrix}.$$

3. Find $det\mathbf{A}^T$, where

$$\mathbf{A} = \begin{pmatrix} 0 & 2 & 0 & 1 \\ 2 & -2 & 1 & -3 \\ 6 & 1 & 10 & 0 \\ -3 & 0 & 4 & 1 \end{pmatrix}.$$

4. The system

$$\begin{aligned} 3x_1 - x_2 + 5x_3 - x_4 &= 0 \\ x_1 + 4x_2 - x_3 + x_4 &= 0 \\ -3x_1 - 7x_3 + 5x_4 &= 0 \end{aligned}$$

has

A. No solutions
B. Infinitely many solutions
C. Exactly one solution

5. Consider the matrix

$$\mathbf{A} = \begin{pmatrix} -1 & 2 & 6 \\ 3 & -1 & 0 \\ 0 & 1 & -2 \end{pmatrix}$$

(a) Find the adjoint of $\mathbf{A}$, $adj(\mathbf{A})$;

(b) Find the inverse of $\mathbf{A}$ by means of the formula

$$\mathbf{A}^{-1} = \frac{1}{det(\mathbf{A})}\, adj(\mathbf{A}).$$

6. Use the method of variation of parameters to find the general solution of the differential equation

$$y'' + 25y = 5\sec(5x).$$

7. Let $\mathbf{A}$ be a 3×3 matrix such that

$$\mathbf{A}\mathbf{A}^T = \begin{pmatrix} 5 & 2 & 5 \\ 2 & 2 & 1 \\ 6 & 1 & 10 \end{pmatrix}.$$

Find $det(\mathbf{A})$.

8. Determine if the following subsets S of $\mathbf{V}$ are subspaces of $\mathbf{V}$. Provide motivation for your answers.

   (i) $\mathbf{V} = M_2(\mathbb{R})$; $S = \{\mathbf{A} \in M_2(\mathbb{R}) \ : \ det(\mathbf{A}) = 1\}$;

   (ii) $\mathbf{V} = M_2(\mathbb{R})$; $S = \left\{\mathbf{A} \in M_2(\mathbb{R}) \ : \ \mathbf{A} = \begin{pmatrix} x & -x \\ y & z \end{pmatrix}\right\}$;

   (iii) $\mathbf{V} = C(\mathbb{R})$; $S = \{f \in C(\mathbb{R}) \ : \ f(0) = f(1)\}$.

9. Consider the three vectors in $\mathbb{R}^3$

$$\mathbf{v_1} = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}, \quad \mathbf{v_2} = \begin{pmatrix} 0 \\ 2 \\ 0 \end{pmatrix}, \quad \mathbf{v_3} = \begin{pmatrix} 0 \\ 3 \\ 2 \end{pmatrix}.$$

Prove that $\{\mathbf{v_1}, \mathbf{v_2}, \mathbf{v_3}\}$ span $\mathbb{R}^3$.

10. Consider the vector space $C^\infty(I)$.
    (a) Prove that the three functions

$$1,\ 3x,\ x^2 - 1$$

    are linearly independent in $C^\infty(I)$.
    (b) Decide whether or not the function $f(x) = 1 + x^2 - 2x$ is in the $span\{1, 3x, x^2 - 1\}$.