Ma 262, Spring 2002

Practice Exam 1

1. Find the general solution of the differential equation

$$y' = \frac{1}{y^{1/2}(1+x^2)}.$$

2. Solve the linear differential equation of the first order

$$y' - (cosx)y = cosx.$$

3. Find the general solution for $x > 0$ of the second order differential equation

$$x^2 y'' = xy' + (y')^3.$$

(**Note**: To calculate an indefinite integral of the type

$$y = \int \frac{x}{\sqrt{a+bx}} dx$$

use the substitution $t = a + bx$)

4. In a culture a certain strain of bacteria is growing at a rate which is proportional to the number present. Initially there were 70 bacteria in the culture. If the number of bacteria after 5 hours is 210 find the doubling time of the strain.

5. Find the general solution of the differential equation

$$y'' - 4y \;=\; e^x \sin(x).$$

6. Determine the motion of the spring-mass system governed by

$$\frac{d^2y}{dt^2} + 5\frac{dy}{dt} + 6y = 0, \qquad y(0) = -1, \qquad \frac{dy}{dt}(0) = 4,$$

and find the time at which the mass passes through the equilibrium position.

7. Find the general solutions of the following differential equations:

$$[e^x(1+x)\sin 2y - 2y\sin x]\,dx + [2xe^x \cos 2y + 2\cos x]\,dy = 0.$$

8. Given that $y_1(t) = t^{-1}$ is a solution of

$$2t^2y'' + 3ty' - y = 0, \qquad t > 0,$$

find a second linearly independent solution.

9. Solve the differential equation

$$\frac{dy}{dx} = \frac{y^2 + 2xy}{x^2}.$$

10. Suppose that the temperature of a cup of coffee obeys Newton's law of cooling. If the coffee has a temperature of 200° F when freshly poured, and one minute later has cooled to 190° F in a room at 70° F, determine when the coffee reaches a temperature of 150° F.