Practice Problems for MA 262 Final, Spring 2009

1. Find the general solution of
   $y' - y\tan x = 8\sin^3 x$

   A. $y = 2\sin^3 x\tan x + C\sec x$ B. $y = \sin^3 x\cos x + C\cos x$ C. $y = 2(\sin x)^{-1} + C$
   D. $y = 2(\sin x)^{-4}\cos x + C\cos x$ E. $y = 2\sin^3 x\tan x + C\cos x$

2. A tank initially holds a solution of 10 lbs. of salt in 100 gallons of water. A salt solution containing 0.2 lbs. of salt per gallon runs into the tank at the rate of 3 gallon per day. Water evaporates from the tank at the rate of 1 gallon per day and the well-stirred solution runs out of the tank at the rate of 2 gallons per day. Let $x(t)$ be the amount of salt in the tank at time t (in days). Find a differential equation for $x(t)$.

   A. $x' = 0.3 - \dfrac{3x}{100}$ B. $x' = 0.2 - \dfrac{2x}{100}$ C. $x' = 0.6 - \dfrac{3x}{100}$
   D. $x' = 0.6 - 3x$ E. $x' = 0.6 - \dfrac{2x}{100}$

3. Find the general solution of the differential equation
   $$\frac{dy}{dx} = -\frac{2xy+1}{x^2+y}$$

   A. $x^2y + x + \frac{1}{2}y^2 = C$ B. $x^2y + x = C$ C. $xy^2 + y + \frac{1}{3}x^3 = C$
   D. $xy + \frac{1}{3}x^3 = C$ E. $xy^2 + y + \frac{1}{2}x^2 = C$

4. The solution $y(4)$ to the initial value problem
   $y'' + y = 0 \qquad y(0) = 1, \qquad y'(0) = 1$
   have value $y'\left(\frac{\pi}{4}\right) =$

   A. $\sqrt{2}$ B. 2 C. 0 D. $2\sqrt{2}$ E. $\frac{\sqrt{2}}{2}$

5. The general solution of $y'' + 2y' + 5y = 0$ is $y =$

   A. $e^{-x}(A\cos x + B\sin x)$ B. $Ae^{-2x} + Bxe^{-2x}$ C. $e^{-x}(A\cos 2x + B\sin 2x)$
   D. $Ae^{3x} + Be^{2x}$ E. $e^{-x}(A\cos\sqrt{2}x + B\sin\sqrt{2}x)$

6. If $y = u_1y_1 + u_2y_2$, where $y_1 = e^x$ and $y_2 = xe^x$ is a particular solution of the equation
   $y'' - 2y' + y = x^2e^x$,
   then $u_1 + u_2 =$

   A. $\dfrac{x^3}{3} + \dfrac{x^4}{4}$ B. $\dfrac{x^3}{3} - \dfrac{x^4}{4}$ C. $x^3 - x^4$ D. $\dfrac{x^2}{2} - \dfrac{x^3}{3}$ E. $x^2 - x^3$

7. The function $y_1 = e^{-x}$ is a solution of the differential equation

$$xy'' + (1+2x)y' + (x+1)y = 0$$

If $y_2 = u(x)e^{-x}$ is the second linearly independent solution, then u satisfies the differential equation

A. $u'' - 2u' + u = 0$ B. $xu''e^{-x} - 2xu' = 0$ C. $u' - u = 0$
D. $xu'' + u' = 0$ E. $xu'' + (1+2x)u' + (x+1)u = 0$

8. The trial solution of

$$y'' - y' - 2y = 4x^3e^{-x}$$

in the method of undetermined coefficient is

A. $e^{-x}(A_0 + A_1x + A_2x^2 + A_3x^3 + A_4x^4)$ B. $xe^{-x}(A_0 + A_1x + A_2x^2 + A_3x^3)$
C. $x^2e^{-x}(A_0 + A_1x + A_2x^2)$ D. $x^3e^{-x}(A_0 + A_1x)$ E. $A_0x^4e^{-x}$

9. Find all values of k so that the following system of equations has no solutions.

$$\begin{cases} x + y - z = 1 \\ 2x + 3y + kz = 2 \\ x + ky + 3z = k - 1 \end{cases}$$

A. $k = -3$ B. $k = 2$ C. $k = 1$ D. $k \neq 2$ E. $k \neq -3$

10. The Wronskian of the function $\{1, x, x^2\}$ is

A. 0 B. 1 C. 5 D. 10 E. 2

11. Determine the value or values of k for which the polynomials $x + x^2$, $1 + x$, $2 + kx^2$ are linearly dependent.
A. $k = 2$ B. $k = 0$ C. $k = -2$ D. $k = 1$ E. $k = -1$

12. Which of the following are vector spaces?
i) the set of all 2×2 non-singular matrices
ii) the set of all continuous functions with $f(a) = f(a + 2\pi)$
iii) the set of all vectors of the form $(r + s, r, r - s)$, r, s real
A. (i) and (ii) B. (i) and (iii) C. (ii) and (iii) D. (i), (ii) and (iv) E. only (iii)

13. Let L be the linear differential operator of order 2 given by $L = D^2 + 5xD - 3$. Compute $L(x)$.
A. $x^2 + 5x - 3$ B. $2x$ C. $x^2 + 5x^2 - 3$ D. $3x^2 + 5x + 1$ E. x^3

14. Let L be the differential operator $L = (D+1)^3$. Compute a basis for the kernel of L.
A. $e^x, \cos x, \sin x$ B. e^x, e^{-x}, xe^x C. $e^{-x}, xe^{-x}, x^2e^{-x}$
D. e^x, e^{-x}, xe^{-x} E. $e^x, e^{-x}\cos x, e^{-x}\sin x$

15. Which of the following are a basis for P_4, the set of all polynomials of degree < 4?

(i) $\{x, x^2, x^3, x^4\}$

(ii) $\{x, x^2+1, x^3+2, x-3\}$

(iii) $\{x+3, x^2+1, x^2-1, x+5\}$

A. (i) and (ii) B. (i) or (ii) C. (ii) and (iii) D. (ii) only E. (iii) only

16. Find the value of k for which the following 4 vectors fail to span R^3;

$$v_1 = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \; v_2 = \begin{bmatrix} 1 \\ -2 \\ 3 \end{bmatrix}. \; v_3 = \begin{bmatrix} 2 \\ 2 \\ 6 \end{bmatrix}, \; v_4 = \begin{bmatrix} 1 \\ 6 \\ k \end{bmatrix}$$

A. $k=0$ B. $k=1$ C. $k=2$ D. $k=3$ E. $k=4$

17. Let $T : R^5 \to R^2$ be the linear transformation given by $T(\mathbf{x}) = A\mathbf{x}$, where

$$A = \begin{bmatrix} 2 & -3 & 4 & -5 & 1 \\ 6 & -9 & 12 & 0 & 0 \end{bmatrix}.$$

Find the dimension of $\ker(T)$.

A. 0 B. 1 C. 2 D. 3 E. 4

18. Let $T : R^2 \to R^2$ be a linear transformation such that $T(1,2) = (1,3)$ and $T(3,2) = (-3,5)$. Then $T(1,0) =$

A. $(-2,1)$ B. $(-4,2)$ C. $(2,8)$ D. $(-1,4)$ E. $(4,2)$

19. The product of the eigenvalues of the matrix $M = \begin{bmatrix} 1 & 2 \\ -1 & 4 \end{bmatrix}$ is

A. 2 B. 3 C. 4 D. 5 E. 6

20. Which of the following matrices are defective?

$$A = \begin{bmatrix} 1 & 2 \\ -2 & 5 \end{bmatrix}, \; B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}, \; C = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 2 \end{bmatrix}$$

A. A only B. B only C. C only D. A and B E. B and C

21. The general solution of the system of first order differential equations

$$\begin{bmatrix} x_1' \\ x_2' \\ x_3' \end{bmatrix} = \begin{bmatrix} 3 & 0 & 0 \\ 1 & 1 & 2 \\ -1 & 2 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

is

A. $C_1 \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix} e^{-3t} + C_2 \begin{bmatrix} -2 \\ 0 \\ 1 \end{bmatrix} e^{3t} + C_3 \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix} e^{-t}$

B. $C_1 \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix} e^{3t} + C_2 \begin{bmatrix} -2 \\ 0 \\ 1 \end{bmatrix} te^{3t} + C_3 \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix} e^{-t}$

C. $C_1 \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix} e^{-3t} + C_2 \begin{bmatrix} -2 \\ 0 \\ 1 \end{bmatrix} e^{-3t} + C_3 \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix} e^{t}$

D. $C_1 \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix} e^{3t} + C_2 \begin{bmatrix} -2 \\ 0 \\ 1 \end{bmatrix} e^{3t} + C_3 \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix} e^{t}$

E. $C_1 \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix} e^{3t} + C_2 \begin{bmatrix} -2 \\ 0 \\ 1 \end{bmatrix} e^{3t} + C_3 \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix} e^{-t}$

22. A 2×2 matrix A has a characteristic value $\lambda = -2+i$ with corresponding characteristic vector $\begin{bmatrix} -1 \\ 1+i \end{bmatrix}$. One real solution of $\mathbf{x}' = A\mathbf{x}$ is

A. $e^{-2t}\cos t \begin{bmatrix} -1 \\ 1 \end{bmatrix} + e^{t}\sin 2t \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

B. $e^{t}\cos 2t \begin{bmatrix} -1 \\ 1 \end{bmatrix} + e^{t}\sin 2t \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

C. $e^{-2t}\cos t \begin{bmatrix} -1 \\ 1 \end{bmatrix} - e^{-2t}\sin t \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

D. $e^{-2t}\cos t \begin{bmatrix} 1 \\ 1 \end{bmatrix} + e^{2t}\sin t \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

E. $e^{t}\cos 2t \begin{bmatrix} -1 \\ 1 \end{bmatrix} - e^{t}\sin 2t \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

23. If a fundamental matrix for $\mathbf{x}' = A\mathbf{x}$ is $X(t) = \begin{bmatrix} e^{-2t} & e^{2t} \\ 0 & 2e^{2t} \end{bmatrix}$, then a particular solution to the nonhomogeneous system $\mathbf{x}' = A\mathbf{x} + \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ is

A. $\begin{bmatrix} e^{-2t} & e^{2t} \\ 0 & 2e^{2t} \end{bmatrix} \begin{bmatrix} e^{2t} \\ 0 \end{bmatrix}$ B. $\begin{bmatrix} e^{-2t} & e^{2t} \\ 0 & 2e^{2t} \end{bmatrix} \begin{bmatrix} \frac{1}{2}e^{2t} \\ 0 \end{bmatrix}$ C. $\begin{bmatrix} e^{2t} & -\frac{1}{2}e^{-2t} \\ 0 & \frac{1}{2}e^{-2t} \end{bmatrix} \begin{bmatrix} \frac{1}{2}e^{2t} \\ 0 \end{bmatrix}$

D. $\begin{bmatrix} e^{2t} & -\frac{1}{2}e^{-2t} \\ 0 & \frac{1}{2}e^{-2t} \end{bmatrix} \begin{bmatrix} \frac{1}{2}e^{2t} \\ 0 \end{bmatrix}$ E. None of the above

24. If

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 2 & -3 & 3 \\ 1 & -1 & 1 \end{bmatrix} \text{ and } B = A^{-1} \text{ then } b_{31} =$$

A. 0 B. 1 C. -1 D. 3 E. -3

25. The matrix $A = \begin{bmatrix} -3 & -1 \\ 2 & -1 \end{bmatrix}$ has an eigenvalue $-2 + i$. An eigenvector of A is

A. $(1 + i, 2)$ B. $(1 - i, 2)$ C. $(1 + i, -2)$ D. $(2 + i, 1)$ E. $(2 - i, 1)$

Answers: 1. A 2. E 3. A 4. C 5. C 6. B 7. D 8. B 9. A
10. E 11. C 12. C 13. B 14. C 15. D 16. D 17. D
18. A 19. E 20. D 21. E 22. C 23. B 24. B 25. C