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Geometric properties of solutions to subelliptic equations in nilpotent Lie groups.

Reaction diffusion systems (*Trieste*, 1995), 89–105, *Lecture Notes in Pure and Appl. Math.*, 194, *Dekker, New York*, 1998.

This paper deals with the study of geometric properties of the level sets of solutions to sub-Laplacians in Heisenberg type groups. In such a group one can define a notion of dilation and consequently a notion of starlikeness with respect to a fixed point. Let Ω be a C^2, starlike (with respect to $e \in \Omega$) subdomain of a Heisenberg type group G. Denote by L the sub-Laplacian associated to G and let u be the unique L-harmonic function in the complement of Ω such that $u = 1$ on $\partial\Omega$, and $\lim_{x\to\infty} u(x) = 0$. The main result of the paper states that all the level sets of u are starlike with respect to e. The techniques involved in the proof are quite delicate. An important role is played by Bony's maximum principle and by some pointwise estimates on the gradient of u. The latter are based on the use of a family of special mollifiers introduced by the reviewer and the authors [Rend. Sem. Mat. Univ. Politec. Torino **51** (1993), no. 4, 361–386 (1994); MR 96b:35030]. One of the main ingredients of the proof consists in the representation formula $u(x) = \mathrm{Cap}(\Omega)\Gamma(x, e)(1+\omega(x))$ in terms of the capacity of Ω, $\mathrm{Cap}(\Omega)$, the fundamental solutions of L, $\Gamma(x, y)$, and a correction factor $\omega(x)$ that decays faster than the inverse of the distance to the origin.

{For the entire collection see 98d:35004}   *Luca Capogna* (1-AR)