95a:46050 46E35 35H05

Danielli, D. (I-BOLO)

A compact embedding theorem for a class of degenerate Sobolev spaces. (English. Italian summary)

Rend. Sem. Mat. Univ. Politec. Torino **49** (1991), no. 3, 399–420 (1993).

Let Ω be a bounded open set in \mathbb{R}^n , and let A(x) $(x \in \Omega)$ be a positive semidefinite $n \times n$ matrix such that $\langle A(x)\xi, \xi \rangle = \sum_1^m \langle X_j(x), \xi \rangle^2$, where X_1, \dots, X_m are Lipschitz continuous vector fields on Ω . For $1 \leq p < \infty$, define the A-Sobolev space $W^p_A(\Omega)$ to be the completion of $C_0^{\infty}(\Omega)$ with respect to the norm $||u||_p + [\int_{\Omega} \langle A \nabla u, \nabla u \rangle^{p/2}]^{1/p}$. The main theorem of this paper is that, under certain natural but somewhat technical geometric hypotheses on A, the inclusion $W^p_A(\Omega) \to L^p(\Omega)$ is compact. It is also shown that the hypotheses are satisfied when A defines a sub-Riemannian structure on Ω in the sense of B. Franchi and E. Lanconelli [J. Math. Pures Appl. (9) **64** (1985), no. 3, 237–256; MR 87g:35099] or when the vector fields X_j satisfy Hörmander's hypoellipticity condition. Gerald B. Folland (1-WA)