On the Structure of positive radial solutions for quasilinear equations

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Abstract

This paper deals with the existence, multiplicity and nonexistence of positive radial solutions to the problem $\mathrm{div}(A(|\nabla u|)\nabla u)+\lambda k(|x|)f(u)=0$ in $R_1 < |x| < R_2,\ x \in \mathbb{R}^n,\ u=0$ on $|x|=R_1$ and $|x|=R_2$. We are particularly interested in the effects of the parameter λ on the problem under a general assumption on the function $A(|p|)$, which covers the two important cases $A \equiv 1$ and $A(|p|)=|p|^{m-2}, m>1$. In this paper we assume that $k, f \in C([0,\infty),[0,\infty))$ and $k(t) \not\equiv 0$ on any subinterval of $[R_1, R_2]$. Let $f_0 = \lim_{u\to 0^+}[\frac{f(u)}{A(u)}]/u$ and $f_\infty = \lim_{u\to\infty}[\frac{f(u)}{A(u)}]/u$. We prove that if either $f_0 = 0$ and $f_\infty = \infty$ (superlinear), or $f_0 = \infty$ and $f_\infty = 0$ (sublinear), then for all $\lambda > 0$ the problem has a positive radial

solution. In addition, we assume that $f(u) > 0$ for $u > 0$. Then either $f_0 = f_\infty = 0$, or $f_0 = f_\infty = \infty$, guarantee the existence of two positive radial solutions for certain intervals of λ. On other hand, either f_0 and $f_\infty > 0$, or f_0 and $f_\infty < \infty$, imply the nonexistence of positive radial solutions for certain intervals of λ. Furthermore, all the results are valid for Dirichlet/Neumann boundary conditions.