

Math 351, Midterm 3

December 3, 2014

Name: _____

This exam consists of six pages including this front page.

Rules

1. You have one hour to complete the exam.
2. No calculators, books, or notes are allowed.
3. Show your work for every problem. Unjustified answers will receive no credit.
4. You may write on the front and back of each page. Extra paper is available.

<i>Score</i>		
1	20	
2	20	
3	20	
4	20	
5	20	
<i>Total</i>	100	

1. Explain why each of the statements is either true or false. If false, please provide a counterexample.
 - (a) Let A be a 3×3 matrix. Then $\det(5A) = 5 \det(A)$.
 - (b) If A is an $n \times n$ matrix such that 0 is an eigenvalue of A , then A cannot be invertible.

2. Suppose the matrix

$$A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}.$$

has determinant $\det A = 7$. Calculate the determinant of the matrix

$$B = \begin{bmatrix} 5g - 4d & 5h - 4e & 5i - 4f \\ 3d - 2a & 3e - 2b & 3f - 2c \\ a & b & c \end{bmatrix}.$$

3. Solve the system of equations

$$\begin{aligned}x + 2y + z &= 1 \\x + 2y + 2z &= 0 \\3x + 3y + z &= 2\end{aligned}$$

using Cramer's rule.

4. Let A be an $n \times n$ matrix with inverse $B = A^{-1}$. State the formula for the value b_{ij} of the i th row and j th column of B . If

$$A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 1 & 2 & 2 & 1 \\ 3 & 5 & 2 & 1 \\ 0 & 2 & 4 & 2 \end{bmatrix}$$

and $B = A^{-1}$, calculate b_{14} and b_{41} .

5. Consider the matrix

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & 2 \\ 0 & 0 & 3 \end{bmatrix}.$$

Calculate the eigenvalues and eigenspaces of A . Use this to find an eigenbasis \mathcal{B} for A , and calculate the product $Q^{-1}AQ$, where $Q = P_{\mathcal{B}}$ is the point matrix for the eigenbasis \mathcal{B} .