

Numerical Methods for Partial Differential Equations

The Fall Semester

1. Two-Point Boundary Value Problems

- finite difference methods
- finite element methods
- linear algebra: Gaussian elimination, and iteration for linear and nonlinear problems

2. Two-Dimensional Elliptic Problems

- finite difference methods: maximum principle and l_2 convergence proofs
- finite element methods: energy and L^2 estimates
- linear algebra: see §4 below.

3. Parabolic Problems

- explicit finite difference methods
- implicit finite difference methods: backward Euler and Crank-Nicolson methods
- finite element methods
- nonlinear problems, with locally linear techniques

4. Iterative Methods (introductory coverage)

- Elliptic Problems
 - (1) Jacobi and Gauss-Seidel
 - (2) SOR and SSOR
 - (3) Alternating direction methods
 - (4) Conjugate gradient methods (preconditioned)
 - (5) Domain decomposition and multigrid methods
- Parabolic Problems: time-stepping implicit methods using fixed number of PCG iterations

5. Wave Equations

The Spring Semester

Several (four to six) of the following topics will be covered:

1. Approximation Theory: Bramble-Hilbert (Dupont-Scott, Jamet)
2. Mixed Methods (elliptic and parabolic)
3. Finite Volume Methods (mixed methods with numerical integration)
4. Domain Decomposition and Multigrid iterations
5. Transport-Dominated Diffusive Systems (first-order hyperbolic)
6. Least-Squares Methods
7. Mesh Refinement
8. Incompressible Stokes and Navier-Stokes Equations
9. Elasticity
10. Pseudo-Differential Equations (Helmholtz, etc.)
11. Maxwell's equations
12. Porous Media problems
13. p and hp -Methods
14. Spectral Methods
15. Boundary Integral Methods

References

- [1] S. C. BRENNER AND L. R. SCOTT, *The Mathematical Theory of Finite Element Methods*, Springer-Verlag, New York, 1994.
- [2] P. G. CIARLET, *The Finite Element Method for Elliptic Problems*, North-Holland, New York, 1978.
- [3] C. JOHNSON, *Numerical Solution of Partial Differential Equations by the Finite Element Method*, Cambridge, Cambridge University Press, 1987.
- [4] A. QUARTERONI AND A. VALLI, *Numerical Approximation of Partial Differential Equations*, Springer-Verlag, Berlin, 1994.
- [5] J. C. STRIKWERDA, *FINITE DIFFERENCE SCHEMES AND PARTIAL DIFFERENTIAL EQUATIONS*, Wadsworth & Brooks, Pacific Grove, California, 1989.