Quiz # 3:

① Given the curve $r(t)=\left(\frac{1}{3}t^3, \frac{1}{2}t^2, 3\right)$. Find $v(t)$, $T(t)$ and $N(t)$ when $t=1$.

Solution:

- $r'(t)=\left(t^2, t, 0\right)=v(t)$; then $v(1)=r'(1)=(1,1,0)$.
- $|r'(t)|=\sqrt{t^4+t^2}$. Now $T(t)=\frac{r'(t)}{|r'(t)|}$; then $T(1)=\frac{r'(1)}{|r'(1)|}=\frac{1}{\sqrt{2}}(1,1,0)$.
- $T(t)=\frac{r'(t)}{|r'(t)|}$; then $T'(t)=\left(\frac{1}{|r'(t)|}\right)' r'(t)+\frac{1}{|r'(t)|} r''(t)$. Now:
- $r''(t)=(2t,1,0)$ and :
- $\left(\frac{1}{|r'(t)|}\right)'=\left(\frac{1}{\sqrt{t^4+t^2}}\right)'=\left(\left(t^4+t^2\right)^{-\frac{1}{2}}\right)'=-\frac{1}{2}\left(t^4+t^2\right)^{-\frac{1}{2}-1}\cdot 4t^3+2t$

$$=-\frac{(2t^3+t)}{\sqrt{(t^4+t^3)^3}}$$

- Then: $r''(1)=(2,1,0)$ and $\left.\left(\frac{1}{|r'(t)|}\right)'\right|_{t=1}=-\frac{3}{\sqrt{2^3}}=-\frac{3}{2\sqrt{2}}$
- $N(1)=\frac{T'(1)}{|T'(1)|}$ where $T'(1)=-\frac{3}{2\sqrt{2}}(1,1,0)+\frac{1}{\sqrt{2}}(2,1,0)$

$$=\left(\frac{1}{2\sqrt{2}}, -\frac{1}{2\sqrt{2}}, 0\right).$$

$|T'(1)|=\sqrt{2\cdot\frac{1}{4\cdot 2}}=\frac{1}{2}$, thus $N(1)=2\left(\frac{1}{2\sqrt{2}}, \frac{-1}{2\sqrt{2}}, 0\right)$

$$=\left(\frac{1}{\sqrt{2}}, \frac{-1}{\sqrt{2}}, 0\right).$$

② Let $v(t) = (t^2, t+1, e^{-t})$ and $r(0) = (2,3,1)$.

Find $r(t)$.

Solution: As $v(t) = r'(t)$; then:

$$r(t) = \int v(t)dt + C_1$$ where C_1 is constant vector; suppose $C_1 = (c_1, c_2, c_3)$

Then

$$\int v(t)dt = \left(\int t^2 dt, \int (t+1)dt, \int e^{-t}dt\right)$$

$$= \left(\frac{t^3}{3}, t^2+t, -e^{-t}\right);$$ thus:

$$r(t) = \left(\frac{t^3}{3}, t^2+t, -e^{-t}\right) + (c_1, c_2, c_3)$$

Now $r(0) = (0,0,-1) + (c_1, c_2, c_3)$ must be $(2,3,1)$; so

$r(0) = (c_1, c_2, c_3 - 1) = (2,3,1)$ then $c_1 = 2$; $c_2 = 3$ and $c_3 - 1 = 1$

that means $c_3 = 2$; therefore:

$$r(t) = \left(\frac{t^3}{3}, t^2+t, -e^{-t}\right) + (2,3,2)$$

$$= \left(\frac{t^3}{3}+2, t^2+t+3, 2-e^{-t}\right).$$