Quiz 3.

TA Xi

Recitation 2:30 3:30

1. For the curve given by $r(t) = \langle \frac{1}{3}t^3, \frac{1}{2}t^2, t \rangle$ Find

a) the unit tangent vector

b) $T'(t)$ when $t = 1$

Solution a) $$T(t) = \frac{r'(t)}{|r'(t)|} = \frac{\langle t^2, t, 1 \rangle}{\sqrt{t^4+t^2+1}}$$

b) $$T'(t) = \left\langle \frac{2t\sqrt{t^4+t^2+1} - \frac{t^2(4t^3+2t)}{2\sqrt{t^4+t^2+1}}}{t^4+t^2+1}, \frac{\sqrt{t^4+t^2+1} - \frac{t(4t^3+2t)}{2\sqrt{t^4+t^2+1}}}{t^4+t^2+1}, \frac{0 - \frac{(4t^3+2t)}{2\sqrt{t^4+t^2+1}}}{t^4+t^2+1} \right\rangle$$

$$T'(1) = \left\langle \frac{2\sqrt{3}-\sqrt{3}}{3}, \frac{\sqrt{3}-\sqrt{3}}{3}, -\frac{\sqrt{3}}{3} \right\rangle$$

$$= \left\langle \frac{\sqrt{3}}{3}, 0, -\frac{\sqrt{3}}{3} \right\rangle$$

2. A particle moves with initial velocity $\vec{v} = \langle t^2, t+1, e^{-t} \rangle$ and $\vec{r}(0) = \langle 2, 3, 1 \rangle$

a) Find the formula for $\vec{r}$ at all time.

b) When the y-component of the velocity is 2, what is $\vec{r}(t)$?

Solution a) $$\vec{r}(t) = \int \vec{v}(t)\,dt = \left\langle \frac{t^3}{3}, \frac{t^2}{2}+t, -e^{-t} \right\rangle + C$$

$$\because \vec{r}(0) = \langle 2, 3, 1 \rangle = \langle 0, 0, -1 \rangle + C$$

$$\therefore C = \langle 2, 3, 2 \rangle \quad \therefore \vec{r}(t) = \left\langle \frac{t^3}{3}+2, \frac{t^2}{2}+t+3, 2-e^{-t} \right\rangle$$

b) $t+1 = 2 \qquad t = 1$

$$\vec{r}(t) = \vec{r}(1) = \left\langle \frac{1}{3}+2, \frac{1}{2}+4, 2-e^{-1} \right\rangle$$